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Frequency response in the parameter plane

Glavis, George Oscar

Monterey, California. Naval Postgraduate School

FREQUENCY RESPONSE IN THE PARAMETER PLANE

GEORGE OSCAR GLAVIS

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PARAMETER PLANE

by

George Oscar Glavis
Lieutenant, United States Navy
B.S., United States Naval Academy, 1961

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ABSTRACT

Algebraic Methods are employed for frequency response studies of physical systems. The transfer function of each system's mathematical model is composed of at least two independent variables embedded in the coefficients. These, together with magnitude, angular frequency, and phase angle are the variable parameters to be studied.

Computer programs are developed for graphically representing these variable parameters. Interpretation of these results is made to evaluate the utility of such methods in design and analysis studies.

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1. Introduction.

The dynamic response of linear systems can be found basically by two methods: experimental and analytical. Continuous advancements in technology have precluded the feasibility of the former, relegating its utility to hybrid studies at best. Increased attention has therefore been focused on the analytical techniques available, on development of new techniques where necessary, or possible improvements in known techniques. A fundamental dilemma to the analytic approach is the synthesis of one or more linearly independent differential equations which adequately describe the system to be studied. Whether the problem consists of multiple equations involving eigen values, or a single mathematical statement, the system limitations and characteristics are revealed by a thorough study of the roots of the characteristic equation. This equation is generated implicitly by matrix representation of a set of simultaneous first order equations, or explicitly by a rearrangement of a single higher order polynomial.

Many sound advances have been made since the turn of the century in the realm of root location techniques for time invariant systems. As control engineering emerged from its embrionic stage and became embedded more firmly in all phases of engineering, added emphasis was placed upon the knowledge of systems behavior in a dynamic atmosphere consisting of at least one varying parameter. Among the more important achievements in system stability analysis are the Routh, Mikhailov, and Nyquist criteria. Frequency response characteristically has been greatly utilized as a fundamental system specification, and the graphical techniques of Bode and later Nichols have become widely used throughout the world for both analysis and design. Many other graphical tools have been formulated, including sophisticated root locus schemes involving cascade compensation, and various forms of feedback. Inherent in all of the above, however, has been the severe restriction of evaluating systems behavior as a function of only one variable parameter. The number of systems which truly fall within this category are in the minority, and it therefore becomes necessary to employ single variable techniques on multivariable problems. The result is a non-optimum compromise between system sensitivity and computational requirements, i.e., the calculations necessary to accurately predict system behavior in multivariable environments become unwieldly.

Several noteworthy achievements have been made to partially overcome this difficulty. Extensive application of the analog-computer has been made to problems suitable for its use. The advent of the digital computer has opened a magnitude of possibilities, one of which has been the emergence of state space theory. A large potential exists in a relatively new field known as Optimal Control Theory, which is founded upon the maximum principal approach of Pontryagen, and Bellman's dynamic programming method. Stochastic processing and predicting filtering theory have become very influential in the field of controls due in large part to the accomplishments of Wiener and Kalman. A still different approach has dealt with the various possible algebraic manipulations of the characteristic equation, hence the name ALGEBRAIC METHODS. It is this topic to which the following study is devoted. Basically, these methods employ techniques which permit engineering systems to be designed and evaluated by simultaneous variation of two parameters. Extension to one or more additional parameter variations is possible, but again the computational requirements and intelligent interpretation of results become dominant factors, just as in the expansion of single parameter techniques. Little imagination is needed to impress in one's mind the impact that this tool will have in optimizing two parameter design problems as its capabilities become better defined, and engineers achieve proficiency in its use.

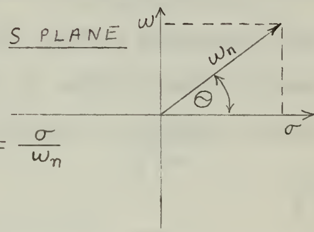
The following section presents the rudimentary mathematical theory as applied to electrical control systems employing two variable parameters. Primary emphasis is placed upon formulating the necessary relations for frequency response graphical representations. Subsequent sections are devoted to the generation and evaluation of these displays from the standpoint of analysis and design. Frequency response and damping criteria are investigated by means of various forms of derivative feedback. Noise limitations in physically realizable systems preclude the usage of derivative feedback in excess of second order in most cases, and for this reason the study of such stabilizing schemes is not considered practical. Typical examples are presented and a comprehensive evaluation of each is endeavored. Graphical representations are utilized and analyzed according to their respective value for information content. The concluding section recapitulates the most important limitations and advantages revealed by each graphical analysis example, and postulates further extensions of specific developments where applicable. A generalized version of computer programs employed, with an explanatory flowgraph of each is included in the Appendix.

2. Mathematical Theory of Algebraic Methods.

Conventional network analysis techniques rely heavily upon pole-zero representations in the complex frequency plane, as well as real time transient and frequency response curves. These methods are not conducive to the synthesis, or design problems, however. System parameters are seldom specified precisely, and a slight variation in one might correspond to an excessive alteration in another while still satisfying yet a third design criterion. The single parameter variation is seldom a rewarding approach to this type of problem, and it therefore becomes necessary to transform response curves into another reference frame where system parameters are the variables. Fundamental complex variable transformation theory provides the necessary conversion media, based upon Cauchy's Principle of Argument.

2.1 Transformation Techniques Employing Chebyshev Polynomials.

The ensuing development is based upon three fundamental steps, and permits transforming the complex frequency plant into a coefficient plane and a parameter plane, where the corresponding variables are arbitrarily designated as a_x, a_y and alpha (α), beta (β) respectively. The first step involves choosing a contour to be transformed. The complex frequency damping constant zeta (ζ) has a radial line contour in the s-plane, and is selected as the transformation contour. A mapping function must be considered next, and for this the characteristic equation is appropriate. The third stipulation requires a transformation variable which allows the mapping function to be represented by two or more independent equations. In this case, it was decided to separate the complex variable into its real and imaginary parts. Listed below are the necessary steps required to achieve this transformation. Considerable attention is devoted to a complete and detailed development in order to provide continuity in the fundamental relationships which might be readily referenced as major divergence points for certain ensuing pursuits. The basic relations presented in the following diagram and adjacent equations serve merely to establish a consistent notation for all further developments. The approach, although exceedingly elementary, is considered necessary to avoid perpetuating existent ambiguities where similar ground rules have not been employed.



$$s = \sigma + j\omega$$

$$\theta = \cos^{-1}(-\zeta) \Longrightarrow \cos\theta = -\zeta = \frac{\sigma}{\omega_n}$$

$$\sigma = \omega_n \cos\theta$$

$$\cos^2\theta = \zeta^2 = 1 - \sin^2\theta$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\zeta^2} = \frac{\omega}{\omega_n}$$

$$\omega = \omega_n\sqrt{1-\zeta^2} = \omega_n\sqrt{1-\cos^2\theta} = \omega_n \sin\theta$$

Substituting the relations for σ and ω into the top equation:

$$s = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2} = \omega_n\cos\theta + j\omega_n\sin\theta$$

$$s = (\cos\theta + j\sin\theta)$$

Employing the results of the mathematician Euler, this last equation may be written: $s = \omega_n e^{j\theta}$

And using DeMoivre's Theorem the relation for any power of s is found to be:

$$s^k = \omega_n^k e^{jk\theta} = \omega_n^k(\cos k\theta + j\sin k\theta) \tag{2.1-1}$$

It is now necessary to define two new variables:

$$T_K(-\zeta) = \cos k\theta = \cos\left(k\cos^{-1}(-\zeta)\right)$$

$$U_K(-\zeta) = \frac{\sin k\theta}{\sin\theta} = \frac{\sin\left(k\cos^{-1}(-\zeta)\right)}{\sin\left(\cos^{-1}(-\zeta)\right)}$$

Substitution of these equivalences into (2.1-1) yields:

$$s^K = \omega_n\left[T_K(-\zeta) + j\sqrt{1-\zeta^2}\,U_K(-\zeta)\right]$$

Since engineering problems involving stable systems are concerned with positive values of zeta, it is advantageous to employ the following relations which are obtained by trigonometric identities:

$$T_K(-\zeta) = -1^K\,T_K(\zeta) \quad ; \quad U_K(-\zeta) = -1^{K+1}\,U_K(\zeta)$$

All characteristic equations can be reduced to a general form which is herein defined as:

$$F(s) = a_0 + a_1 s + a_2 s^2 + \cdots a_{n-1} s^{n-1} + a_n s^n = 0$$

Rewriting this in summation notation, and employing the identities developed above, we can write:

$$F(s) = \sum_{K=0}^{n} a_K s^K = \sum_{K=0}^{n} a_K \sigma^K + j\sum_{K=0}^{n} a_K \omega^K = 0$$

$$F(s) = \sum_{K=0}^{n} a_K \omega_n^K \cos k\theta + j\sum_{K=0}^{n} a_K \omega_n^K \sin k\theta = 0$$

$$F(s) = \sum_{K=0}^{n} a_K \omega_n^K (-1)^K T_K(\zeta) + j\sum_{K=0}^{n} a_K \omega_n^K (-1)^{K+1} U_K(\zeta) = 0$$

A polynomial may be set to zero when its orthogonal components are independently zero; hence:

$$\sum_{K=0}^{n} a_K \omega_n^K (-1)^K T_K(\zeta) = 0 \qquad (2.1\text{-}2)$$

$$\sum_{k=0}^{n} a_k w_n^{\,k} (-1)^{k+1} U_k(\xi) = 0 \tag{2.1-3}$$

It would now seem plausible to inspect several values of these new variables $T_k(\xi)$ and $U_k(\xi)$, in order that any discernable characteristic traits might be utilized.

$$T_0(\xi) = (-1)^0 T_0(-\xi) = T_0(-\xi) = \cos(0\theta) = \cos(0) = 1$$

$$T_1(\xi) = (-1)^1 T_1(-\xi) = -T_1(-\xi) = \cos(\cos^{-1}(-\xi)) = \xi$$

$$U_0(\xi) = (-1)^1 U_0(-\xi) = \frac{-\sin(0\theta)}{\sin(\theta)} = \frac{-0}{\sin(\theta)} = 0$$

$$U_1(\xi) = (-1)^2 U_1(-\xi) = \frac{\sin(\cos^{-1}(-\xi))}{\sin(\cos^{-1}(-\xi))} = 1$$

Comparison of these polynomials with a table of Chebyshev functions will reveal their identity. Accordingly, the following recurrence relations may be employed for complete tabulation.

$$T_{k+1}(\xi) - 2\xi T_k(\xi) + T_{k-1}(\xi) = 0$$

$$U_{k+1}(\xi) - 2\xi U_k(\xi) + U_{k-1}(\xi) = 0$$

It can be shown that $T_k(\xi)$ and $U_k(\xi)$ are related by the following:

$$T_k(\xi) = \xi U_k(\xi) - U_{k-1}(\xi)$$

Substituting this into (2.1-2) yields:

$$\sum_{k=0}^{n} a_k w_n^{\,k} (-1)^k [\xi U_k(\xi)] - \sum_{k=0}^{n} a_k w_n^{\,k} (-1)^k [U_{k-1}(\xi)] = 0 \tag{2.1-4}$$

which can be rewritten as follows:

$$\zeta \sum_{K=0}^{n} a_K \omega_n{}^K (-1)^K [U_K(\zeta)] - \sum_{K=0}^{n} a_K \omega_n{}^K (-1)^K [U_{n-1}(\zeta)] = 0$$

Factoring a (-1) out of (2.1-3) and multiplying that result through by (ζ) in a separate step yields the following sequential equations, so arranged for easy inspection.

$$\sum_{K=0}^{n} a_K \omega_n{}^K (-1)^K U_K(\zeta) = 0 = \zeta \left[\sum_{K=0}^{n} a_K \omega_n{}^K (-1)^K U_K(\zeta) \right]$$

This last form of (2.1-3) is identically equal to the first term of (2.1-4). Therefore the two fundamental relations become:

$$\sum_{K=0}^{n} (-1)^K a_K \omega_n{}^K U_{K-1}(\zeta) = 0 \qquad (2.1\text{-}5)$$

$$\sum_{K=0}^{n} (-1)^K a_K \omega_n{}^K U_K(\zeta) = 0 \qquad (2.1\text{-}6)$$

The utility of (2.1-5) and (2.1-6) is not obvious, but their judicious use results in a powerful tool. Employment of the bilateral transformation allows the natural angular frequency (ω_n) to range from minus infinity to plus infinity, thereby eliminating it as an unknown variable per se. Choosing a specific value for zeta (ζ) allows the values of $U(\zeta)$ to be obtained by the preceding method or by reference to precomputed tables [1]. Thus any two values of a_K can be allowed to vary, thereby formulating two independent equations in terms of two variable coefficients of the characteristic equation. Mitrovic [2], who pioneered the above development, chose the two lowest order coefficients, a_0 and a_1 , for his study. Elliott, Heseltine, and Thaler, [3] and summarily Siljak [4] extended Mitrovic's approach to additional coefficients. A brief summary of the coefficient plane derivation appears on the next page.

2.2 Coefficient Plane

For the generalized form of the characteristic equation, let two variable coefficients be defined as a_x and a_y. The order of the equation (n) is no smaller than x, and x is greater than y. The value of y is non-negative. $(n \geq x > y \geq 0)$ Transposing the a_x and a_y terms in (2.1-5) and (2.1-6) yields the following results:

$$(-1)^x a_x w_n^x U_{x-1}(\xi) + (-1)^y a_y w_n^y U_{y-1}(\xi) = \sum_{\substack{k=0 \\ k \neq x \\ k \neq y}}^{n} (-1)^k a_k w_n^k U_{k-1}(\xi)$$

$$(-1)^x a_x w_n^x U_x(\xi) + (-1)^y a_y w_n^y U_y(\xi) = \sum_{\substack{k=0 \\ k \neq x \\ k \neq y}}^{n} (-1)^k a_k w_n^k U_k(\xi)$$

Solving the above equations simultaneously for a_x and a_y yields:

$$a_x = \sum_{\substack{k=0 \\ k \neq x \\ k \neq y}}^{n} (-1)^{k-x} w^{k-x} a_k \left[\frac{U_{y-1}(\xi)\, U_k(\xi) - U_y(\xi)\, U_{k-1}(\xi)}{U_y(\xi)\, U_{x-1}(\xi) - U_{y-1}(\xi)\, U_x(\xi)} \right] \quad (2.2\text{-}1)$$

$$a_y = \sum_{\substack{k=0 \\ k \neq x \\ k \neq y}}^{n} (-1)^{k-y} \omega^{k-y} a_k \left[\frac{U_{y-1}(\xi) U_k(\xi) - U_y(\xi) U_{k-1}(\xi)}{U_x(\xi) U_{y-1}(\xi) - U_{x-1}(\xi) U_y(\xi)} \right] \quad (2.2\text{-}2)$$

It will now be shown that:

$$U_{x-y}(\xi) = U_y(\xi) U_{x-1}(\xi) - U_{y-1}(\xi) U_x(\xi) \quad (2.2\text{-}3)$$

Let $x = 2$

$$U_{2-y}(\xi) = U_1(\xi) U(\xi) - U_2(\xi) U_{y-1}(\xi)$$

From the preceding Chebyshev recursion relations:

$$U_1(\xi) = 1$$

$$U_2(\xi) = 2\xi U_1(\xi) - U_0(\xi) = 2\xi(1) - 0 = 2\xi$$

Therefore:

$$U_{2-y}(\xi) = U_y(\xi) - 2\xi U_{y-1}(\xi) \quad (2.2\text{-}4)$$

But it has been shown that:

$$U_{-k}(\xi) = -U_k(\xi) \quad (2.2\text{-}5)$$

Substituting (2.2-5) into (2.2-4), we see the general recursion formula as previously stated for Chebyshev polynomials:

$$U_y(\xi) - 2\xi U_{y-1}(\xi) + U_{y-2}(\xi) = 0$$

It can be shown by inductive reasoning that the above relation exists for all values of K [5]. Therefore we can also write:

$$U_{K-y}(\xi) = U_y(\xi)\, U_{K-1}(\xi) - U_{y-1}(\xi)\, U_K(\xi)$$

Thus the general solution for two variable coefficients becomes:

$$a_x = - \sum_{\substack{K=0 \\ K \neq x \\ K \neq y}}^{n} (-1)^{K-x} \omega^{K-x} a_K \frac{U_{K-y}(\xi)}{U_{x-y}(\xi)}$$

$$a_y = \sum_{\substack{K=0 \\ K \neq x \\ K \neq y}}^{n} (-1)^{K-y} \omega^{K-y} a_K \frac{U_{K-x}(\xi)}{U_{x-y}(\xi)}$$

A graphical display of the characteristic equation is often formed with a_x as one axis and a_y as the other axis. Such a plot is termed a coefficient plane. With sufficient data and interpolation, it is possible to derive a very satisfactory design in this manner for systems whose characteristic equations possess the desired variables in coefficient form. An excellent treatise on the applied coefficient plane can be found in the NASA Contractor Report by Thaler, Siljak, and Dorf [6].

2.3 Parameter Plane, Linear Case.

The coefficient plane provides a means of working with two variable coefficients of a polynomial, as long as each of these variables does not appear in more than one coefficient. It can be recalled that each coefficient within a polynomial is merely the sum of the product of the roots taken m at a time, where m equals the order of the equation minus the order of the particular coefficient's independent variable. Therefore severe restrictions are encountered when attempting to use this method for simultaneous, but independent variation of any two roots of a polynomial. Thus the coefficient plane method does not include the more general class of problems which have the desired variables embedded within several coefficients. Siljak [7] postulated that this class of problems could be handled in a manner similar to that of the coefficient plane, provided these variables appeared linearly. Designating the variable parameters as alpha (α) and beta (β), one can rewrite any coefficient of a general polynomial as follows:

$$a_K = b_K \alpha + c_K \beta + d_K$$

Using this expanded form, the two independent equations (2.1-5) and (2.1-6) become:

$$\alpha \sum_{K=0}^{n} (-1)^K b_K \omega_n^K U_{K-1}(\xi)$$

$$+ \beta \sum_{K=0}^{n} (-1)^K c_K \omega_n^K U_{K-1}(\xi)$$

$$+ \sum_{K=0}^{n} (-1)^K d_K \omega_n^K U_{K-1}(\xi) = 0$$

$$\alpha \sum_{k=0}^{n} (-1)^{k} b_k w_n^{k} U_k(\xi) + \beta \sum_{k=0}^{n} (-1)^{k} c_k U_k(\xi)$$

$$+ \sum_{k=0}^{n} (-1)^{k} d_k w_n^{k} U_k(\xi) = 0$$

This rather unwieldy notation will be slightly modified by the following substitutions:

Let:

$$B_1(\xi, w_n) = \sum_{k=0}^{n} (-1)^{k} b_k w_n^{k} U_{k-1}(\xi)$$

$$C_1(\xi, w_n) = \sum_{k=0}^{n} (-1)^{k} c_k w_n^{k} U_{k-1}(\xi)$$

$$D_1(\xi, w_n) = \sum_{k=0}^{n} (-1)^{k} d_k w_n^{k} U_{k-1}(\xi)$$

$$B_2(\xi, w_n) = \sum_{k=0}^{n} (-1)^{k} b_k w_n^{k} U_k(\xi)$$

$$C_2(\xi, w_n) = \sum_{k=0}^{n} (-1)^{k} c_k w_n^{k} U_k(\xi)$$

$$D_2(\xi, w_n) = \sum_{k=0}^{n} (-1)^{k} d_k w_n^{k} U_k(\xi)$$

Employment of these substitutions reduces (2.1-5) and (2.1-6) to the following form:

$$\alpha B_1(\xi, w_n) + \beta C_1(\xi, w_n) + D_1(\xi, w_n) = 0$$

$$\alpha B_2(\xi, w_n) + \beta C_2(\xi, w_n) + D_2(\xi, w_n) = 0$$

These two equations are quite easily solved for alpha and beta by Cramer's rule, or by employing the algebraic method of elimination. The resultant solutions are:

$$\alpha = \frac{C_1(\xi,\omega_n)\,D_2(\xi,\omega_n) - C_2(\xi,\omega_n)\,D_1(\xi,\omega_n)}{B_1(\xi,\omega_n)\,C_2(\xi,\omega_n) - B_2(\xi,\omega_n)\,C_1(\xi,\omega_n)} \qquad (2.3\text{-}1)$$

$$\beta = \frac{D_1(\xi,\omega_n)\,B_2(\xi,\omega_n) - D_2(\xi,\omega_n)\,B_1(\xi,\omega_n)}{B_1(\xi,\omega_n)\,C_2(\xi,\omega_n) - B_2(\xi,\omega_n)\,C_1(\xi,\omega_n)} \qquad (2.3\text{-}2)$$

The stability variables most commonly employed by control engineers are the damping ratio, zeta, and the undamped natural frequency, (ω_n). Most of the current literature places great importance upon these terms, and interprets system performance by correlation with the standardized second order system. In some cases, however, the rectangular complex frequency plane configuration consisting of the imaginary frequency sigma (σ) and the real frequency omega (ω) is more useful than the polar coordinate system with damping ratio, zeta (ξ) and the undamped natural frequency (ω_n). In second order system parlance, sigma represents the rate of exponential decay of the response signal, while omega is the angular frequency at which this signal oscillates. The necessary steps for representing (2.3-1) and (2.3-2) in terms of sigma and omega involve the use of the binomial expansion. A brief outline of this method is now presented. Recalling the basic relations between σ, ω, ξ and ω_n in 2.1, it is possible to rewrite (2.1-1) as follows:

$$s^K = \omega_n{}^K e^{jK\theta} = (\sigma + j\omega)^K = X_K + jY_K$$

where the individual components of X_K and Y_K are seen to have a particular recurrence pattern as shown by the following tabulation:

$$s^0 = 1$$
$$s^1 = \sigma + j\omega$$
$$s^2 = \sigma^2 - \omega^2 + 2j\omega$$
$$s^3 = \sigma^3 - 3\sigma\omega^2 + j(3\sigma^2\omega - \omega^3)$$
$$s^4 = \sigma^4 - 6\sigma^2\omega^2 + \omega^4 + j(4\sigma^3\omega - 4\sigma\omega^3)$$
$$s^5 = \sigma^5 - 10\sigma^3\omega^3 + 5\sigma\omega^4 + j(5\sigma^4\omega - 10\sigma^2\omega^3 + \omega^5)$$

From which:

$$X_0 = 1$$
$$X_1 = \sigma$$
$$X_2 = \sigma^2 - \omega^2$$
$$X_3 = \sigma^3 - 3\sigma\omega^2$$
$$X_4 = \sigma^4 - 6\sigma^2\omega^2 + \omega^4$$
$$Y_0 = 0$$
$$Y_1 = \omega$$
$$Y_2 = 2\sigma\omega$$
$$Y_3 = 3\sigma^2\omega - \omega^3$$
$$Y_4 = 4\sigma^3\omega - 4\sigma\omega^3$$

which can be consolidated into binomial expansion form for the generalized case.

$$s^K = \left[\binom{K}{0}\sigma^K - \binom{K}{2}\sigma^{K-2}\omega^2 + \binom{K}{4}\sigma^{K-4}\omega^4 - \cdots \right]$$
$$+ j\left[\binom{K}{1}\sigma^{K-1}\omega - \binom{K}{3}\sigma^{K-3}\omega^3 + \cdots \right] = 0$$

$$S^k = \sum_{i=0}^{k} \left\{ \left[(-1)^i \binom{k}{2i} \sigma^{k-2i} \omega^{2i} \right] \right.$$

$$\left. + j \left[(-1)^{i+1} \binom{k}{2i+1} \sigma^{k-2i+1} \omega^{2i+1} \right] \right\} = 0$$

where $\binom{k}{x} = \dfrac{k!}{(k-x)!\,(x)!}$ for $k \geq x$

and $\binom{k}{x} = 0$ whenever $k < x$

Recalling the generalized form of the characteristic equation $F(s) = \sum_{k=0}^{n} a_k s^k$, we can again separate s^k into its perpendicular components x_k and y_k such that $\sum_{k=0}^{n} a_k x_k$ represents the summation of the real terms, and $\sum_{k=0}^{n} a_k y_k$ denotes the summation of all imaginary terms. Thus:

$$\sum_{k=0}^{n} a_k x_k = \sum_{k=0}^{n} \sum_{i=0}^{k} a_k \left[(-1) \binom{k}{2i} \sigma^{k-2i} \omega^{2i} \right]$$

$$\sum_{k=0}^{n} a_k y_k = \sum_{k=0}^{n} \sum_{i=0}^{k} a_k \left[(-1) \binom{k}{2i+1} \sigma^{k-2i+1} \omega^{2i+1} \right]$$

In order to effectively utilize these equations for computer applications, it is necessary that the recursion relation be explicitly formulated. This can be achieved by observing the following:

$$s = \sigma + j\omega \implies s - \sigma - j\omega = 0$$

$$s = \sigma - j\omega \implies s - \sigma + j\omega = 0$$

multiplying these two equations;

$$(s - \sigma - j\omega)(s - \sigma + j\omega) = 0$$

Expanding this, we have:

$$s^2 - 2\sigma s + \sigma^2 + \omega^2 = 0$$

Pre-multiplying this by s^k we may write

$$s^k \left[s^2 - 2\sigma s + \sigma^2 + \omega^2 \right] = 0$$

which, upon expansion, becomes

$$s^{k+2} - 2\sigma s^{k+1} + \sigma^2 s^k + \omega^2 s^k = 0$$

This may be rewritten in terms of orthogonal components where

$$s^{k+2} = X_{k+2} + j\,Y_{k+2}$$

$$s^{k+1} = X_{k+1} + j\,Y_{k+1}$$

$$s^{k} = X_{k} + j\,Y_{k}$$

$$X_{k+2} + j\,Y_{k+2} - 2\sigma(X_{k+1} + j\,Y_{k+1}) + (\sigma^2 + \omega^2)(X_k + j\,Y_k) = 0$$

Separating this into its real and imaginary parts results in

$$X_{k+2} - 2\sigma X_{k+1} + (\sigma^2 + \omega^2)\,X_k = 0$$

$$Y_{k+2} - 2\sigma Y_{k+1} + (\sigma^2 + \omega^2)\,Y_k = 0$$

Noting that σ is X_1, and ω is Y_1, we can make these substitutions, resulting in the following explicit recurrence relations:

$$X_{k+2} - 2X_1 X_{k+1} + (X_1^2 + Y_1^2)\,X_k = 0$$

$$Y_{k+2} - 2X_1 Y_{k+1} + (X_1^2 + Y_1^2)\,Y_k = 0$$

Thus one can solve a given coefficient equation with coefficients of the form $a_k = b_k\alpha + c_k\beta + d_k$ for the parameters alpha and beta in terms of the variables sigma and omega. The method is the same as for the case involving the variables zeta and ω_n. The two independent equations become:

$$\alpha \sum_{k=0}^{n} b_k X_k + \beta \sum_{k=0}^{n} c_k X_k + \sum_{k=0}^{n} d_k X_k = 0$$

$$\alpha \sum_{k=0}^{n} b_k Y_k + \beta \sum_{k=0}^{n} c_k Y_k + \sum_{k=0}^{n} d_k Y_k = 0$$

which may be written in abbreviated form as

$$\alpha B_1(\sigma, \omega) + \beta C_1(\sigma, \omega) + D_1(\sigma, \omega) = 0$$
$$\alpha B_2(\sigma, \omega) + \beta C_2(\sigma, \omega) + D_2(\sigma, \omega) = 0$$

where:

$$B_1 = \sum_{K=0}^{n} b_K X_K \qquad B_2 = \sum_{K=0}^{n} b_K Y_K$$

$$C_1 = \sum_{K=0}^{n} c_K X_K \qquad C_2 = \sum_{K=0}^{n} c_K Y_K$$

$$D_1 = \sum_{K=0}^{n} d_K X_K \qquad D_2 = \sum_{K=0}^{n} d_K Y_K$$

As in the preceding development, the solutions for alpha and beta take the form:

$$\alpha = \frac{C_1(\sigma, \omega)\, D_2(\sigma, \omega) - C_2(\sigma, \omega)\, D_1(\sigma, \omega)}{B_1(\sigma, \omega)\, C_2(\sigma, \omega) - B_2(\sigma, \omega)\, C_1(\sigma, \omega)} \qquad (2.2\text{-}3)$$

$$\beta = \frac{D_1(\sigma, \omega)\, B_2(\sigma, \omega) - D_2(\sigma, \omega)\, B_1(\sigma, \omega)}{B_1(\sigma, \omega)\, C_2(\sigma, \omega) - B_2(\sigma, \omega)\, C_1(\sigma, \omega)} \qquad (2.3\text{-}4)$$

These equations are seen to be of the same form as (2.3-1) and (2.3-2). Thus the method of solution is identical to the one using Chebyshev polynomials, with only the recurrence relation and the solution variables being different.

An analogous approach to this problem does not involve the Chebyshev polynomials method at all, as can be seen by the following procedure which starts by employing (2.1-1) directly:

$$F(s) = \sum_{K=0}^{n} a_K s^K = \sum_{K=0}^{n} a_K \omega_n{}^K \cos k\theta$$

$$+ j \sum_{K=0}^{n} a_K \omega_n{}^K \sin k\theta = 0$$

Separating this polynomial into its orthogonal components we may write:

$$Re\left\{F(S)\right\} = \sum_{K=0}^{n} a_K \; \omega_n^{K} \cos k\theta = 0$$

$$Im\left\{F(S)\right\} = \sum_{K=0}^{n} a_K \; \omega_n^{K} \sin k\theta = 0$$

Now substituting the integral components of a_K into each of the above yields the following general terms:

$$\alpha \sum_{K=0}^{n} b_K \omega_n^{K} \cos k\theta + \beta \sum_{K=0}^{n} c_K \omega_n^{K} \cos k\theta + \sum_{K=0}^{n} d_K \omega_n^{K} \cos k\theta = 0$$

$$\alpha \sum_{K=0}^{n} b_K \omega_n^{K} \sin k\theta + \beta \sum_{K=0}^{n} c_K \omega_n^{K} \sin k\theta + \sum_{K=0}^{n} d_K \omega_n^{K} \sin k\theta = 0$$

Solving these equations simultaneously for alpha and beta, we have:

$$\alpha = \frac{\sum_{K=0}^{n} c_K \omega_n^{K} \cos k\theta \sum_{K=0}^{n} d_K \omega_n^{K} \sin k\theta - \sum_{K=0}^{n} c_K \omega_n^{K} \sin k\theta \sum_{K=0}^{n} d_K \omega_n^{K} \cos k\theta}{\sum_{K=0}^{n} b_K \omega_n^{K} \cos k\theta \sum_{K=0}^{n} c_K \omega_n^{K} \sin k\theta - \sum_{K=0}^{n} b_K \omega_n^{K} \sin k\theta \sum_{K=0}^{n} c_K \omega_n^{K} \sin k\theta} \quad (2.3\text{-}5)$$

$$\beta = \frac{\sum_{K=0}^{n} d_K \omega_n^{K} \cos k\theta \sum_{K=0}^{n} b_K \omega_n^{K} \sin k\theta - \sum_{K=0}^{n} d_K \omega_n^{K} \sin k\theta \sum_{K=0}^{n} b_K \omega_n^{K} \cos k\theta}{\sum_{K=0}^{n} b_K \omega_n^{K} \cos k\theta \sum_{K=0}^{n} c_K \omega_n^{K} \sin k\theta - \sum_{K=0}^{n} b_K \omega_n^{K} \sin k\theta \sum_{K=0}^{n} c_K \omega_n^{K} \cos k\theta} \quad (2.3\text{-}6)$$

These two alternative parametric solutions are seen to be no less complicated in form than their Chebyshev polynomial or binomial expansion counterparts. In fact their application is more cumbersome since there is no analogous recursion equation for the trigonometric functions. Therefore the first approach resulting in equations (2.3-1) and (2.3-2) is utilized in all but the most specialized cases since it was the first method to be successfully developed, and its application is more familiar than the sigma-omega form.

<u>EXAMPLE 1</u> (FEEDBACK COMPENSATION FOR LINEAR THIRD ORDER SYSTEM)

At this point it will be advantageous to select a basic system for initial studies. The linear case is sufficiently unencumbered by mathematical manipulations to properly illustrate certain fundamental characteristics, and therefore serves as an appropriate introduction to the forthcoming discussions of section 4. Consider an uncompensated system with a forward transfer function of third order, whose gain is sufficiently large to make it unstable. Such a system is depicted by the unity feedback block diagram and characteristic equation shown in figure 2-1. The corresponding root locus diagram is included in figure 2-2 and it is seen by Routh's Criterion that the limit of stability is at K = 30,000. For larger gains than this critical value, it will be necessary to introduce either cascade or feedback compensation to regain system stability. Selection of tachometer and acceleration feedback results in the compensated block diagram and characteristic equation shown in figure 2-3. Choosing a forward gain value of 100,000 results in a highly unstable uncompensated system. The stabilizing effect of either one of these forms of derivative feedback without the other is shown by the cross hatched root relocation zones of figure 2.4. The effects of simultaneous variation of both forms cannot be as easily predicted on the root locus plane. At best, one can generate several root locus families, each as a function of only one variable parameter, and gain from these patterns an approximate idea of the required parameter values needed to place the dominant roots of the system in a prescribed position. The parameter plane technique becomes a very useful tool in this type of situation, however, since the prescribed root location automatically specifies the required value of zeta and also of ω_n. A cursory inspection of equations (2.3-1) through (2.3-4) will reveal that the following curves can be drawn in the parameter plane, which by convention has alpha on the abcissa and beta on the ordinate:

1. Constant zeta curves, in which zeta is held fixed at a specified value, and ω_n is allowed to vary through a predetermined range.
2. Constant undamped frequency or "natural frequency" (ω_n) curves in which ω_n is held at a specified fixed value, and zeta is allowed to vary from 0 to 1.
3. Constant $\zeta - \omega_n$ curves, in which zeta and ω_n are both varied such that their product is always constant at preselected curve values.
4. Constant sigma (σ) curves, which represent lines parallel to the imaginary axis in the complex frequency plane.
5. Constant "damped frequency" (ω) which represent lines parallel to the real axis in the complex frequency plane, and are constructed by holding ω fixed, and varying sigma from zero through a pre-selected negative value range.
6. Constant real root ($\sigma + j0$) curves, which represent specified fixed points on the negative real axis of the

complex frequency plane. (Thus it is seen that certain points on the s-plane are transformed into lines within the parameter plane. This factor plays a predominant role in singular line theory).

Not all of these curves need be used by any means. The combination of any two of the first five curves positively identifies a complex root in the left half of the complex frequency plane, while the sixth type of curve locates the real roots on the negative sigma axis of the s-plane. Figure 2-5 is one type of parameter plane representation, in which K_a = alpha and K_t = beta. It shows the constant zeta, constant ω_n and real root curves. One can see that for dominant roots at $S = -16.5 \pm j33$ the required values of alpha and beta will be at the intersection of the $\zeta = 0.5$ and $\omega_n = 33$ curves. In this example, they intersect at alpha = 0.0005 and beta = 0.036. The third root is real and can be determined by the real root curve which coincides with these values of alpha and beta. It is seen that the real root is approximately at S = -80. These values are indicated by point A on figure 2-5.

The curves for this example are deceivingly simple, and therefore easy to interpret. Additional care must be employed in higher order equations, where all the root locations are not so easily defined. In order to ensure the desired dominant characteristics, one must certify the location of all existent roots. In exceedingly complex cases, this might necessitate a second, enlarged parameter plane plot centered at the point in question. This, however, is a greatly simplified task compared to single parameter techniques.

2.4 Paramater Plane, Non-Linear Case.

Equations (2.2-6) and (2.2-7) have been developed to handle two variables when they are individual coefficients of a polynomial. Equations (2.3-1) and (2.3-2), or similar forms, apply to two variables embedded linearly in the polynomial equation. The following development is concerned with non-linear arrangements of two variables within the coefficients of a given polynomial. Consider first the case where the variables appear in product form. This arrangement is termed the alpha - beta product case, because the coefficients of the polynomial can be represented as

$$a_K = b_K \alpha + c_K \beta + d_K \alpha\beta + e_K$$

Substituting this into (2.1-5) and (2.1-6), one can write:

$$\alpha \sum_{K=0}^{n} (-1)^K b_K \omega_n^K U_{K-1}(\xi) + \beta \sum_{K=0}^{n} (-1)^K c_K \omega_n^K U_{K-1}(\xi) + \alpha\beta \sum_{K=0}^{n} (-1)^K d_K \omega_n^K U_{K-1}(\xi) + \sum_{K=0}^{n} (-1)^K e_K \omega_n^K U_{K-1}(\xi) = 0$$

$$\alpha \sum_{K=0}^{n} (-1)^K b_K \omega_n^K U_K(\xi) + \beta \sum_{K=0}^{n} (-1)^K c_K \omega_n^K U_K(\xi) + \alpha\beta \sum_{K=0}^{n} (-1)^K d_K \omega_n^K U_K(\xi) + \sum_{K=0}^{n} (-1)^K e_K \omega_n^K U_K(\xi) = 0$$

Employing notational substitutions similar to those in the preceding section, we may write:

$$B_1 = \sum_{K=0}^{n} (-1)^K b_K \omega_n^K U_{K-1}(\xi)$$

$$B_2 = \sum_{K=0}^{n} (-1)^K b_K \omega_n^K U_K(\xi)$$

$$C_1 = \sum_{K=0}^{n} (-1)^K c_K \omega_n^K U_{K-1}(\xi)$$

$$C_2 = \sum_{K=0}^{n} (-1)^K c_K \omega_n^K U_K(\xi)$$

$$D_1 = \sum_{K=0}^{n} d_K \, \omega_n^{K} \, U_{K-1}(\xi)$$

$$D_2 = \sum_{K=0}^{n} d_K \, \omega_n^{K} \, U_K(\xi)$$

$$E_1 = \sum_{K=0}^{n} e_K \, \omega_n^{K} \, U_{K-1}(\xi)$$

$$E_2 = \sum_{K=0}^{n} e_K \, \omega_n^{K} \, U_K(\xi)$$

thereby reducing the preceding equations to the form

$$\alpha B_1(\xi, \omega_n) + \beta C_1(\xi, \omega_n) + \alpha\beta D_1(\xi, \omega_n) + E_1(\xi, \omega_n) = 0$$

$$\alpha B_2(\xi, \omega_n) + \beta C_2(\xi, \omega_n) + \alpha\beta D_2(\xi, \omega_n) + E_2(\xi, \omega_n) = 0$$

The functional subscripts have been included up to this point to emphasize the applicable dependent variables. It will now be necessary to eliminate this descriptive subscript notation in order to achieve better comprehension of mathematical manipulations. Henceforth the dependent variables will be regarded as zeta and ω_n unless otherwise specified, and their inherent inclusion will be assumed in the remainder of this section. Accordingly, the above equations may be rewritten as

$$\alpha B_1 + \beta C_1 + \alpha\beta D_1 + E_1 = 0$$

$$\alpha B_2 + \beta C_2 + \alpha\beta D_2 + E_2 = 0$$

The solution of these simultaneous equations for either alpha or beta is possible provided their Jacobian is not zero, where the Jacobian is denoted:

$$J = (B_1 + \beta D_1)(C_2 + \alpha D_2) - (C_1 + \alpha D_1)(B_2 + \beta C_2)$$

Presuming this to be the case, the solution for beta is found as follows:

$$[B_2 + \beta D_2][\alpha B_1 + \beta C_1 + E_1] = 0$$

$$[-B_1 - \beta D_1][\alpha B_2 + \beta C_2 + E_2] = 0$$

$$[C_1 D_2 - C_2 D_1]\beta^2 + [C_1 B_2 - C_2 B_1 + E_1 D_2 - E_2 D_1]\beta + E_1 B_2 - E_2 B_1 = 0$$

which may be written in determinant notation as

$$\Delta_{CD}\beta^2 + (\Delta_{CB} + \Delta_{ED})\beta + \Delta_{EB} = 0$$

where

$$\Delta_{CD} = \begin{vmatrix} C_1 & C_2 \\ D_1 & D_2 \end{vmatrix} \qquad \Delta_{CB} = \begin{vmatrix} C_1 & C_2 \\ B_1 & B_2 \end{vmatrix}$$

and so forth. Employing the quadratic formula, beta is found to be

$$\beta = -\frac{(\Delta_{CB} + \Delta_{ED}) \pm \sqrt{(\Delta_{CB} + \Delta_{ED})^2 - 4(\Delta_{CD}\Delta_{EB})}}{2\Delta_{CD}} \tag{2.4.1}$$

Proceeding in a similar fashion, it is possible to solve for alpha by the following process:

$$[C_2 + \alpha D_2][\alpha B_1 + (C_1 + \alpha D_1)\beta + E_1] = 0$$

$$[-C_1 - \alpha D_1][\alpha B_2 + (C_2 + \alpha D_2)\beta + E_2] = 0$$

from which:

$$(B_1 D_2 - B_2 D_1)\alpha^2 + (B_1 C_2 - B_2 C_1 + E_1 D_2 - E_2 D_1)\alpha + E_1 C_2 - E_2 C_1 = 0$$

or in determinant notation:

$$\Delta_{BD}\,\alpha^2 + (\Delta_{BC} + \Delta_{ED})\alpha + \Delta_{EC} = 0$$

The desired solution for alpha then becomes

$$\alpha = \frac{-(\Delta_{BC} + \Delta_{ED}) \pm \sqrt{(\Delta_{BC} + \Delta_{ED})^2 - 4(\Delta_{BD}\Delta_{EC})}}{2\,\Delta_{BD}} \qquad (2.4\text{-}2)$$

While the above solutions can be used to obtain alpha and beta, it is advantageous to simplify them for adaptation to computerized programming. Analyzing the terms under the radical signs, it is possible to show their equivalence as follows:

$$\sqrt{(\Delta_{BC} + \Delta_{ED})^2 - 4(\Delta_{BD}\Delta_{EC})} = \sqrt{(\Delta_{CB} + \Delta_{ED})^2 - 4(\Delta_{CD}\Delta_{EB})}$$

Noting that $\Delta_{CB} = -\Delta_{BC}$, and squaring both sides yields

$$(\Delta_{BC} + \Delta_{ED})^2 - 4(\Delta_{BD}\Delta_{EC}) = (-\Delta_{BC} + \Delta_{ED})^2 - 4(\Delta_{CD}\Delta_{EB})$$

which, upon expansion, becomes

$$(\Delta_{BC})^2 + 2(\Delta_{BC}\Delta_{ED}) + (\Delta_{ED})^2 - 4\Delta_{BD}\Delta_{EC} =$$

$$(\Delta_{BC})^2 - 2(\Delta_{BC}\Delta_{ED}) + (\Delta_{ED})^2 - 4\Delta_{CD}\Delta_{EB}$$

Transposing and collecting terms

$$4\Delta_{BC}\Delta_{ED} - 4\Delta_{BD}\Delta_{EC} + 4\Delta_{CD}\Delta_{EB} = 0$$

Dividing through by 4, and expanding determinants,

$$(B_1C_2 - B_2C_1)(E_1D_2 - E_2D_1) - (B_1D_2 - B_2D_1)(E_1C_2 - E_2C_1)$$

$$+ (C_1D_2 - C_2D_1)(E_1B_2 - E_2B_1) = 0$$

$$B_1C_2D_2E_1 - B_2C_1D_2E_1 - B_1C_2D_1E_2 + B_2C_1D_1E_2 - B_1C_2D_2E_1 + B_2C_2D_1E_1$$

$$+ B_1C_1D_2E_2 - B_2C_1D_1E_2 + B_2C_1D_2E_1 - B_2C_2D_1E_1 - B_1C_1D_2E_2 + B_1C_2D_1E_2 = 0$$

Thus, we may define the terms under both radicals as R^2, where

$$R = \sqrt{(\Delta_{BC} + \Delta_{ED})^2 - 4(\Delta_{BD}\Delta_{EC})} = \sqrt{(\Delta_{CB} + \Delta_{ED})^2 - 4\Delta_{CD}\Delta_{EB}}$$

and the solutions for alpha and beta become

$$\alpha = \frac{-(\Delta_{BC} + \Delta_{ED}) \pm R}{2\Delta_{BD}}$$

$$\beta = \frac{-(-\Delta_{BC} + \Delta_{ED}) \pm R}{2\Delta_{CD}}$$

of the four possible combinations for alpha and beta, it is necessary to determine which of these satisfy the original simultaneous equations. The following results are obtained:

$$D_2[\,B_1\alpha + C_1\beta + D_1\alpha\beta + E_1\,] = 0$$

$$-D_1[\,B_2\alpha + C_2\beta + D_2\alpha\beta + E_2\,] = 0$$

$$(B_1D_2 - B_2D_1)\alpha + (C_1D_2 - C_2D_1)\beta + E_1D_2 - E_2D_1 = 0$$

In determinant form this may be written:

$$\Delta_{BD}\alpha + \Delta_{CD}\beta + \Delta_{ED} = 0$$

Substituting the above relations for alpha and beta

$$-\frac{(\Delta_{BC} + \Delta_{ED}) \pm R}{2} - \frac{(-\Delta_{BC} + \Delta_{ED}) \pm R}{2} + \Delta_{ED} = 0$$

Therefore, the radical coefficients for one term, either alpha or beta, must be the negative of the radical coefficient for the other term. This permits the following solutions for alpha and beta:

$$\alpha_K = -\frac{(\Delta_{ED} + \Delta_{BC}) + (-1)^K R}{2\Delta_{BD}} \quad ; \quad k = 1, 2 \qquad (2.4\text{-}3)$$

$$\beta_K = -\frac{(\Delta_{ED} - \Delta_{BC}) + (-1)^K R}{2\Delta_{CD}} \quad ; \quad k = 1, 2 \qquad (2.4\text{-}4)$$

SINGULAR POINTS

Inspection of (2.4-3 and (2.4-4) reveals that limitations are encountered whenever Δ_{BD} or Δ_{CD} equal zero. For these cases, it is necessary to resort to the original set of simultaneous equations for the complementary solution as follows:

Should Δ_{BD} be zero, the two solutions for beta are obtained from (2.4-4) and then substituted into the original simultaneous equations where the solution for alpha is obtained by the following relationship:

$$\alpha_K = \frac{\beta_K C_1 + E_1}{\beta_K D_1 + B_1} = \frac{\beta_K C_2 + E_2}{\beta_K D_2 + B_2} \;; \quad k=1,2 \qquad (2.4\text{-}5)$$

When Δ_{CD} equals zero, the analogous approach yields two solutions for alpha from (2.4-3) which are likewise substituted into the original equations, yielding the following values of beta:

$$\beta_K = \frac{\alpha_K B_1 + E_1}{\alpha_K C_1 + D_1} = \frac{\alpha_K B_2 + E_2}{\alpha_K C_2 + D_2} \;; \quad k = 1,2 \qquad (2.4\text{-}6)$$

SINGULAR LINES

The solution of a set of m simultaneous equations in n unknowns is not always obtainable. When a solution does indeed exist, it sometimes possesses unusual characteristics in that it is not a unique solution. These cases have been catagorized as singular lines, and their applications to parameter plane theory are presented here for completeness. The following development is included for the purpose of showing the necessary and sufficient conditions for singular lines.

Any set of simultaneous equations may be represented in the following matrix notation:

$$[A][X] = [B]$$

where $[A]$ is the coefficient matrix of order $m \times n$

$[X]$ is the variable matrix of order $n \times 1$

$[B]$ is the constant matrix of order $m \times 1$

and $[AB]$ is the augmented matrix.

Any set of simultaneous equations is said to be consistent, (i.e., a solution is guaranteed to exist) when the rank of the coefficient matrix and the rank of the augmented matrix are equal. A unique solution may be found only when the rank of the matrix equals the number of unknowns, n. Otherwise the matrix is said to be singular. These three statements are very subtle, because their simplicity belies their importance. As an illustration of their application, consider the linear second order equations which were derived in the preceding section for coefficients of the form:

$$Re\{F(s)\} = B_1 \alpha + C_1 \beta + D_1 = 0 \tag{2.4-7}$$

$$Im\{F(s)\} = B_2 \alpha + C_2 \beta + D_2 = 0 \tag{2.4-8}$$

from which:

$$\alpha = \frac{C_1 D_2 - C_2 D_1}{B_1 C_2 - B_2 C_1} = \frac{X}{Z} \quad ; \quad \beta = \frac{D_1 C_2 - D_2 C_1}{B_1 C_2 - B_2 C_1} = \frac{Y}{Z}$$

Thus for any characteristic equation of the form

$$\sum_{k=0}^{n} a_k s^k = 0$$

where the coefficients are linear combinations of alpha and beta, it is possible to construct a planar representation as a function of the three terms, X, Y and Z, with alpha and beta as the variable parameters. The inconsistency arises when the denominator determinant Z is zero, and when at least one of the numerator determinants is not zero. Since division of a finite number by zero yields an infinite (or undefined) quotient, it is readily seen that the two simultaneous equations yield two parallel lines, which by definition are everywhere equidistant and therefore never intersect. Thus an inconsistent set of simultaneous equations has no solution. A unique solution is obtained whenever the Z term, or denominator coefficient is not zero. This is the most general type of solution, denoting the intersection of two lines at a finite point in the alpha - beta plane. Should one or both of the numerator determinants X and Y be zero, it merely places the unique value on one or both axes respectively. The latter case of course is just a solution at the origin. The third classification for simultaneous equation solutions is the singular case which exists if and only if all three determinants X, Y, and Z are identically zero. Such a case can exist when the number of unknowns in a set of equations exceeds its rank. Thus one or more of the equations is said to be linearly dependent. Geometrically this represents two collinear lines which by definition intersect everywhere, thereby giving an infinite number of solutions. Extensive value can be gained by the use of singular lines for design purposes, especially in self adaptive systems. It therefore becomes highly desirable to be able to recognize the existence of such lines when depicting a system characteristic equation in the parameter plane. The theory of linear dependence stipulates that two linearly dependent equations differ only by a constant. Thus we may say that for (2.4-7) and (2.4-8) to be linearly dependent, the following conditions must hold:

$$B_2 = k B_1$$

$$C_2 = k C_1$$

$$D_2 = k D_1$$

Because of the exceptionally involved calculations using the Chebyshev polynomial method or the binomial expansion method, the trigometric solutions (2.3-5) and (2.3-6) are amenable to singular line synthesis developments. Employing those equations, (2.4-7) and (2.4-8) can be broken down into the following equivalences:

$$k B_1 = k \sum_{i=0}^{n} b_i \omega_n^i \cos i\theta = B_2 = \sum_{i=0}^{n} b_i \omega_n^i \sin i\theta$$

$$k C_1 = k \sum_{i=0}^{n} c_i \omega_n^i \cos i\theta = C_2 = \sum_{i=0}^{n} c_i \omega_n^i \sin i\theta$$

$$k D_1 = k \sum_{i=0}^{n} d_i \omega_n^i \cos i\theta = D_2 = \sum_{i=0}^{n} d_i \omega_n^i \sin i\theta$$

Using this criterion, it is possible to define the required coefficient values necessary to give singular line solutions, or to determine if any such combinations are indeed possible for a given polynomial order.

Various extensions can be made to include all forms of non-linear polynomials. Such applications merely increase the number of trigonometric equivalence equations, and involve conic sections rather than planar surfaces, provided the independent variables are limited to second order. For higher orders, no geometrical interpretation is possible; however the fundamental theory remains unchanged.

SINGULAR SOLUTIONS

While most equations have explicit solutions, certain differential equations cannot be explicitly defined by specifying the values of the arbitrary constants. In Laplace or Fourier transform theory, this corresponds to non-parametric equations. The solutions to equations written in this form are called singular solutions [8]. Certain forthcoming studies can be facilitated by expressing solutions for alpha and beta in non-parametric form. Up to this point, the simultaneous equations have been written as:

$$X_1(\alpha, \beta, \xi, \omega_n) = 0$$

$$X_2(\alpha, \beta, \xi, \omega_n) = 0$$

with their associated solutions for alpha and beta written as

$$\alpha = A(\xi, \omega_n)$$

$$\beta = B(\xi, \omega_n)$$

The following theory, therefore, is applicable to reducing the above equations into the form:

$$Y(\alpha, \beta, \xi) = 0$$

with associated solutions

$$\alpha = C(\beta, \xi)$$

$$\beta = D(\alpha, \xi)$$

and the alternate form

$$Z(\alpha, \beta, \omega_n) = 0$$

with the solutions

$$\alpha = E(\beta, \omega_n)$$

$$\beta = F(\alpha, \omega_n)$$

It will now be shown that for all integers j and K,

$$\sum_{K=0}^{n} (-1)^K a_K \omega_n^K U_{K-j}(\xi) = 0 \qquad (2.4\text{-}7)$$

whenever the relations (2.1-5) and (2.1-6) can be employed. That this is the case may be proved as follows: Given any polynomial of the form

$$\sum_{K=0}^{n} (-1)^K a_K \omega_n^K U_K(\xi) = 0$$

it is possible to multiply both sides by $U_{j-1}(\xi)$:

$$\sum_{K=0}^{n} (-1)^K a_K \omega_n^K U_{j-1}(\xi) U_K(\xi) = 0 \qquad (2.4\text{-}8)$$

Given also that

$$\sum_{k=0}^{n} (-1)^k a_k \omega_n^k U_{k-1}(\xi) = 0$$

Multiply this by $U_j(\xi)$ to obtain:

$$\sum_{k=0}^{n} (-1)^k a_k \omega_n^k U_j(\xi) U_{k-1}(\xi) = 0$$

Subtracting (2.4-8) from this yields:

$$\sum_{k=0}^{n} (-1)^k a_k \omega_n [U_j(\xi) U_{k-1}(\xi) - U_{j-1}(\xi) U_k(\xi)] = 0$$

Recalling (2.2-3) we may rewrite the above in the following form:

$$\sum_{k=0}^{n} (-1)^k a_k \omega_n^k [U_{k-j}(\xi)] = 0 \qquad (2.4\text{-}7)$$

With the aid of this relationship, it is possible to generate additional equations which will enable (2.1-5) and (2.1-6) to be written in non-parametric form.

To derive non-parametric equations for constant zeta loci, it is necessary to eliminate ω_n from the generalized simultaneous equations which then become

$$Y(\alpha, \beta, \xi) = 0$$

The necessary steps involve Sylvester's Dialytic Method of Elimination [9] for two polynomial equations of different order. An alternate approach, restricted to equations of equal order, can be used with Bezout's Method of Elimination but is not presented here [10]. Consider the following two polynomial equations of order m and n where m may not necessarily be equal to n, and where the coefficients of omega are functions of alpha, beta, and zeta.

$$Y_1(\omega_n) = f_0 + f_1\omega_n + f_2\omega_n^2 + \cdots + f_{m-1}\omega_n^{m-1} + f_m\omega_n^m$$
$$= \sum_{K=0}^{m} f_K(\alpha,\beta,\xi)\,\omega_n^K = 0$$
$$Y_2(\omega_n) = g_0 + g_1\omega_n + g_2\omega_n^2 + \cdots + g_{n-1}\omega_n^{n-1} + g_n\omega_n^n$$
$$= \sum_{K=0}^{n} g_K(\alpha,\beta,\xi)\,\omega_n^K = 0$$

It will be necessary to form a square coefficient matrix out of the combined equations to enable the determinant to be taken. This determinant is then set equal to zero in order to obtain the non-parametric solution for all cases but the trival one which occurs when ω_n simply reduces to zero. The square matrix is of order $m \times n$ and is formed by writing $Y_1(\omega_n)$ and $Y_2(\omega_n)$ as follows:

$$Y_3(\omega_n) = \sum_{j=0}^{n-1} \omega_n^j\left[Y_1(\omega_n)\right] = \sum_{j=0}^{n-1}\sum_{K=0}^{m} f_K(\alpha,\beta,\xi)\,\omega_n^K\,\omega_n^j = 0 \quad (2.4\text{-}8)$$

$$Y_4(\omega_n) = \sum_{j=0}^{m-1} \omega_n^j\left[Y_2(\omega_n)\right] = \sum_{j=0}^{m-1}\sum_{K=0}^{n} g_K(\alpha,\beta,\xi)\,\omega_n^K\,\omega_n^j = 0 \quad (2.4\text{-}9)$$

This corresponds to multiplying each term in $Y_1(\omega_n)$ by $1, \omega_n, \omega_n^2, \ldots \omega_n^{n-3}, \omega_n^{n-2}, \omega_n^{n-1}$ respectively.

In a similar fashion each term in $Y_2(\omega_n)$ is multiplied by $1, \omega_n, \omega_n^2, \ldots \omega_n^{m-1}$ respectively. The matrix equivalent is shown for amplification on the next page.

Equations $Y_1(\omega_n)$ and $Y_2(\omega_n)$ were not constrained to be of equal order, so that the preceding steps might show the method to be applicable for unequal cases. Practical use of this procedure for the elimination of ω_n in context with the parameter plane, however, results in two

$$
Y(\omega) = \begin{bmatrix}
0 & 0 & 0 & f_0 & f_1 & f_2 & \cdot & \cdot & f_{m-2} & f_{m-1} & f_m \\
0 & 0 & f_0 & f_1 & f_2 & \cdot & \cdot & f_{m-2} & f_{m-1} & f_m & 0 \\
0 & f_0 & f_1 & f_2 & \cdot & \cdot & f_{m-2} & f_{m-1} & f_m & 0 & 0 \\
f_0 & f_1 & f_2 & \cdot & \cdot & f_{m-2} & f_{m-1} & f_m & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & g_0 & g_1 & g_2 & \cdot & g_n \\
0 & 0 & 0 & 0 & 0 & g_0 & g_1 & g_2 & \cdot & g_n & 0 \\
0 & 0 & 0 & 0 & g_0 & g_1 & g_2 & \cdot & g_n & 0 & 0 \\
0 & 0 & 0 & g_0 & g_1 & g_2 & \cdot & g_n & 0 & 0 & 0 \\
0 & 0 & g_0 & g_1 & g_2 & \cdot & g_n & 0 & 0 & 0 & 0 \\
0 & g_0 & g_1 & g_2 & \cdot & g_n & 0 & 0 & 0 & 0 & 0 \\
g_0 & g_1 & g_2 & \cdot & g_n & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\omega^0 \\ \omega^1 \\ \omega^2 \\ \omega^3 \\ \omega^4 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \omega^{m+n}
\end{bmatrix} = 0 \qquad \text{(2.4-10)}
$$

equations of equal order when the following substitutions are made for f_K and g_K :

$$f_K = (-1)^K a_K \; U_K(\xi)$$

$$g_K = (-1)^K a_K \; U_{K-n}(\xi)$$

The non-parametric equation for constant ω_n is derived by tactics analogous to those used to generate the constant zeta equation. Again, a set of equations is obtained for the purpose of taking the determinant of the coefficient matrix which, by definition, must be a square matrix if the determinant is to exist. Since the variable matrix $U_K(\xi)$ is only of the first power, it is possible to generate $n-1$ additional equations by use of (2.4-7) which was previously derived. Letting j range from zero to $n-1$ in (2.4-7), the following set of simultaneous equations is obtained:

$$\sum_{k=0}^{n} (-1)^k \quad a_k \quad w_n^{\,k} \quad U_k(\xi) = 0$$

$$\sum_{k=0}^{n} (-1)^k \quad a_k \quad w_n^{\,k} \quad U_{k-1}(\xi) = 0$$

$$\sum_{k=0}^{n} (-1)^k \quad a_k \quad w_n^{\,k} \quad U_{k-2}(\xi) = 0$$

$$\sum_{k=0}^{n} (-1)^k \quad a_k \quad w_n^{\,k} \quad U_{k-3}(\xi) = 0$$

$$\sum_{k=0}^{n} (-1)^k \quad a_k \quad w_n^{\,k} \quad U_{k-4}(\xi) = 0$$

(2.4-11)

$$\vdots$$

$$\sum_{k=0}^{n} (-1)^k \quad a_k \quad w_n^{\,k} \quad U_{k-(n-3)}(\xi) = 0$$

$$\sum_{k=0}^{n} (-1)^k \quad a_k \quad w_n^{\,k} \quad U_{k-(n-2)}(\xi) = 0$$

$$\sum_{k=0}^{n} (-1)^k \quad a_k \quad w_n^{\,k} \quad U_{k-(n-1)}(\xi) = 0$$

Writing these equations in expanded form:

$$(-1)^0 a_0 w_n^{\,0} U_0(\xi) + (-1)^1 a_1 w_n^{\,1} U_1(\xi) + (-1)^2 a_2 w_n^{\,2} U_2(\xi) + \cdots (-1)^n a_n w_n^{\,n} U_n(\xi) = 0$$

$$(-1)^0 a_0 w_n^{\,0} U_{-1}(\xi) + (-1) a_1 w_n U_0(\xi) + (-1)^2 a_2 w_n^{\,2} U_1(\xi) + \cdots (-1) a_n w_n^{\,n} U_{n-1}(\xi) = 0$$

$$(-1)^0 a_0 w_n^{\,0} U_{-2}(\xi) + (-1) a_1 w_n U_{-1}(\xi) + (-1)^2 a_2 w_n^{\,2} U_0(\xi) + \cdots (-1) a_n w_n^{\,n} U_{n-2}(\xi) = 0$$

$$\vdots$$

$$(-1)^0 a_0 w_n^{\,0} U_{-(n-1)}(\xi) + (-1) a_1 w_n U_{-(n-2)}(\xi) + \cdots$$

$$\cdots + (-1)^n a_n w_n^{\,n} U_1(\xi) = 0$$

These equations may be simplified by substituting the following relations:

$$U_0(\xi) = 0$$

$$U_{-K}(\xi) = -U_K(\xi)$$

Thus we may write:

$$\begin{array}{llll}
0 & +(-1)\,a_1\,w_n\,U_1(\xi) & \cdots & +(-1)^n a_n\,w_n^n\,U_n(\xi) = 0 \\
-(-1)^0 a_0\,w_n^0\,U_1(\xi) & +\quad 0 & \cdots & +(-1)^n a_n\,w_n^n\,U_{n-1}(\xi) = 0 \\
-(-1)^0 a_0\,w_n^0\,U_2(\xi) & -(-1)\,a_1\,w_n\,U_1(\xi) & \cdots & +(-1)^n a_n\,w_n^n\,U_{n-2}(\xi) = 0 \\
\vdots & & & \\
-(-1)^0 a_0\,w_n^0\,U_{n-1}(\xi) & -(-1)\,a_1\,w_n\,U_{n-2}(\xi) & \cdots & +(-1)^n a_n\,w_n^n\,U_1(\xi) = 0
\end{array}$$

Which is seen to be a set of n-1 equations with n-1 unknowns. In matrix notation these become:

$$\begin{bmatrix}
(-1)^1 a_1 w_n^1 & +(-1)^2 a_2 w_n^2 + \cdots (-1)^n a_n w_n^n \\
(-1)^2 a_2 w_n^2 - (-1)^0 a_0 w_n^0 & +(-1)^3 a_3 w_n^3 + \cdots \\
(-1)^3 a_3 w_n^3 - (-1)^1 a_1 w_n^1 & +(-1)^4 a_4 w_n^4 - (-1)^0 a_0 w_n^0 + \cdots \\
(-1)^4 a_4 w_n^4 - (-1)^2 a_2 w_n^2 & +(-1)^5 a_5 w_n^5 - (-1)^1 a_1 w_n^1 + \cdots \\
\vdots & \\
(-1)^n a_n w_n^n - (-1)^{n-2} a_{n-2} w_n^{n-2} + \cdots & (-1)^0 a_0 w_n^0
\end{bmatrix}
\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ \vdots \\ U_{n-1} \end{bmatrix} = 0 \qquad (2.4\text{-}12)$$

The above developments have shown how a set of two n^{th} order linearly independent equations with four variables can be reduced to one equation of order n-1 with only three variables. This is accomplished by solving the coefficient matrices of (2.4-10) and (2.4-12) which eliminate zeta and omega respectively. The application of these methods will be indicated in the following article where the mathematical concepts of the frequency response curves are presented.

2.5 Frequency Response Applications of Parameter Plane Theory

Frequency response curves can be portrayed in many ways. Some of the more useful ones will be considered in sections 3 and 4. The various representations of frequency response curves can be derived from the following fundamental theory relating magnitude to frequency. Consider again the transfer function of a mathematical model which represents the system to be studied. This can be reduced to a ratio of two polynomials by partitioning, or other suitable means to arrive at:

$$T(s) = \frac{a_0 + a_1 s + a_2 s^2 + a_3 s^3 + \cdots a_n s^n}{b_0 + b_1 s + b_2 s^2 + b_3 s^3 + \cdots b_m s^m} = \frac{\sum_{i=0}^{n} a_i s^i}{\sum_{j=0}^{m} b_j s^j} \equiv \frac{N(s)}{D(s)} \qquad (2.5\text{-}1)$$

To find the absolute magnitude of this transfer function, the following is noted:

$$M^2 \equiv \left| T(j\omega) \right|^2 = \left[T(j\omega) \right] \left[T(-j\omega) \right]$$

(Although the notation standards of section 2 clearly state the distinction between ω and ω_n, it is important to note that natural angular frequency is no longer being used.) The following steps will illustrate this transition:

$$T(j\omega) = \frac{a_0 + a_1 j\omega + a_2 (j\omega)^2 + a_3 (j\omega)^3 + \cdots a_n (j\omega)^n}{b_0 + b_1 j\omega + b_2 (j\omega)^2 + b_3 (j\omega)^3 + \cdots b_n (j\omega)^n}$$

Rewriting this to separate the real and imaginary parts, we have:

$$T(j\omega) = \frac{a_0 + a_2 (j\omega)^2 + a_4 (j\omega)^4 + \cdots + a_1 j\omega + a_3 (j\omega)^3 + \cdots}{b_0 + b_2 (j\omega)^2 + b_4 (j\omega)^4 + \cdots + b_1 j\omega + b_3 (j\omega)^3 + \cdots}$$

$$T(j\omega) = \frac{[a_0 - a_2 \omega^2 + a_4 \omega^4 - \cdots] + j[a_1 \omega - a_3 \omega^3 + a_5 \omega^5 \ldots]}{[b_0 - b_2 \omega^2 + b_4 \omega^4 - \cdots] + j[b_1 \omega - b_3 \omega^3 + b_5 \omega^5 \ldots]}$$

In abbreviated notation, this becomes:

$$T(j\omega) = \frac{N_{even}(j\omega) + N_{odd}(j\omega)}{D_{even}(j\omega) + D_{odd}(j\omega)}$$

Proceding in a similar manner we find the following:

$$T(-j\omega) = \frac{[a_0 - a_2 \omega^2 + a_4 \omega^4 - \cdots] - j[a_1 \omega - a_3 \omega^3 + a_5 \omega^5 \ldots]}{[b_0 - b_2 \omega^2 + b_4 \omega^4 - \cdots] - j[b_1 \omega - b_3 \omega^3 + b_5 \omega^5 \ldots]}$$

Employing abbreviated notation again, we may write:

$$T(-j\omega) = \frac{N_{even}(j\omega) - N_{odd}(j\omega)}{D_{even}(j\omega) - D_{odd}(j\omega)}$$

Thus the square of the absolute magnitude is found to be:

$$M^2 = \frac{[N_{even}(j\omega) + N_{odd}(j\omega)][N_{even}(j\omega) - N_{odd}(j\omega)]}{[D_{even}(j\omega) + D_{odd}(j\omega)][D_{even}(j\omega) - D_{odd}(j\omega)]}$$

$$M^2 = \frac{[N_{even}(j\omega)]^2 - [N_{odd}(j\omega)]^2}{[D_{even}(j\omega)]^2 - [D_{odd}(j\omega)]^2} \qquad (2.5\text{-}2)$$

$$M^2 = \frac{[a_0 - a_2 \omega^2 + a_4 \omega^4 - a_6 \omega^6 + \cdots]^2 - j^2 [a_1 \omega - a_3 \omega^3 + a_5 \omega^5 \ldots]^2}{[b_0 - b_2 \omega^2 + b_4 \omega^4 - b_6 \omega^6 + \cdots]^2 - j^2 [b_1 \omega - b_3 \omega^3 + b_5 \omega^5 \ldots]^2}$$

This can be written in the following summation notation:

$$M^2 = \frac{\left[\sum_{\substack{K=0\\K\ \text{even}}}^{2n} (-1)^{\frac{K}{2}} c_K w^K\right] + \left[\sum_{\substack{K=1\\K\ \text{odd}}}^{2n-1} (-1)^{\frac{K-1}{2}} c_K w^K\right]}{\left[\sum_{\substack{\ell=0\\\ell\ \text{even}}}^{2m} (-1)^{\frac{\ell}{2}} d_\ell w^\ell\right] + \left[\sum_{\substack{\ell=1\\\ell\ \text{odd}}}^{2m-1} (-1)^{\frac{\ell-1}{2}} d_\ell w^\ell\right]}$$

$$M^2 = \frac{\sum_{K=0}^{n} (-1)^k c_{2K} w^{2k}}{\sum_{\ell=0}^{m} (-1)^\ell d_{2\ell} w^{2\ell}}$$

Let

$X = w^2$

$e_K = c_{2k}$

$f_\ell = d_{2\ell}$

Substituting these new variables above, a more concise notation may be formalized for easier illustration of pertinent relationships.

$$M^2 = \frac{\sum_{K=0}^{n} (-1)^K e_K X^k}{\sum_{\ell=0}^{m} (-1)^\ell f_\ell X^\ell} \tag{2.5-3}$$

In general, the coefficients of (2.5-1) are non-linear combinations of at least two variable parameters, designated alpha and beta. The linear combinations were discussed in article 2.3 and the associated form of any polynomial coefficient was found to be

$a_K = b_K \alpha + c_K \beta + d_K$

For the non-linear case, the simplest combination was seen to be the alpha-beta product combination. This subject was treated in article 2.4 where the resultant coefficients of a given polynomial were broken down in the following manner:

$$a_K = b_K \alpha + c_K \beta + d_K \alpha\beta + e_K$$

Theoretically, there is no limit to the degree of difficulty which exists in the non-linear case. A few examples of the complexity involved in non-linear parameter combinations are presented here for completeness.

$$a_K = b_K \alpha + c_K \beta + d_K \alpha\beta + e_K + f_K \alpha^2 + g_K \beta^2$$

$$a_K = b_K \alpha + c_K \beta + d_K \alpha\beta + e_K + f_K \alpha^2 + g_K \beta^2 + h_K \alpha\beta^2 + i_K \alpha^2\beta + j_K \alpha^3 + l_K \beta^3$$

$$a_K = b_K \alpha + c_K \beta + d_K \alpha\beta + e_K + f_K \alpha^2 + g_K \beta^2 + h_K \alpha\beta^2 + i_K \alpha^2\beta + j_K \alpha^3 + l_K \beta^3 + m_K \alpha\beta^3 + n_K \alpha^2\beta^3 + o_K \alpha^3\beta^2 + p_K \alpha^3\beta + q_K \alpha^4 + r_K \beta^4$$

Clearly, it can be seen that such combinations are exceedingly unwieldy. Attempts to handle all but the simplest arrangements are extremely laborious. Computer programming is still in the embrionic stage for parameter plane applications. Attempts to utilize this tool have been extended only to the alpha-beta non-linear case. This restriction is acute when considering the frequency response curves. A cursory review of the mathematical developments at the beginning of this article will show that the polynomial coefficients of (2.5-3) are obtained by squaring the polynomial coefficients of (2.5-1). Thus, even the simple linear case of article 2.3 results in non-linear combinations of second order for frequency response studies. To handle this problem, it becomes necessary to depart from the procedures which were employed in previous developments. By assigning a value to either alpha or beta, the task is reduced to merely a quadratic formula solution for the remaining unknown variable. This can be illustrated in the following manner: Given a non-linear coefficient combination of the alpha - beta product form:

$$a_K = b_K \alpha + c_K \beta + d_K \alpha\beta + e_K$$

Assign a value to alpha, and denote the value as A:

$$a_K = b_K A + c_K \beta + d_K A \beta + e_K$$

This may be rearranged for clarity,

$$a_K = [\, b_K A + e_K \,] + [\, c_K + d_K A \,] \beta \tag{2.5-4}$$

where the first term on the right hand side denotes a constant value, and the second bracketed term represents a coeficient of the independent variable beta. In a similar manner, a fixed value may be assigned to beta. For illustration purposes, let it be denoted as B. This gives:

$$a_K = b_K \alpha + c_K B + d_K \alpha B + e_K$$

which is likewise rewritten as:

$$a_K = [\, c_K B + e_K \,] + [\, b_K + d_K B \,] \alpha \tag{2.5-5}$$

Again, the first bracketed term represents a constant while the second denotes the coefficient of the independent variable alpha. The computer program for frequency response curves, program PARAMS, is based upon this solution procedure. Consider first the case where magnitude and omega are specified. By assigning incremental values to either alpha or beta, the corresponding solution may be obtained for the other. This is the method employed in the graphical representations which have both alpha and beta on the axes. For the graphs where magnitude is an axis variable, it is incremented while a corresponding value of beta is obtained as a function of alpha and omega. The same philosophy is utilized when omega is an axis variable. For the case where both magnitude and omega are on the axes, the values of alpha and beta are pre-selected and the familiar Bóde plot results. All of these graphical arrangements are treated with considerable detail in section 3. Their introduction at this point was simply to point out that the solutions are based on (2.5-3) and either (2.5-4) or (2.5-5). Since (2.5-3) is the fundamental gain versus frequency equation, it can be anticipated that all frequency response parameter plots will contain the same basic information as a series of conventional Bode frequency response curves. When two variable parameters alpha and beta are utilized together with magnitude and omega, the problem becomes one of four dimensional space. When still another polynomial coefficient variable parameter is considered, an additional dimension is added. In spite of this, it is still possible to derive one set of parameter curves from another, since each is just a different projection from the same multi-dimensional model. The value of each separate representation lies in the ability to depict certain important characteristics which would not be obvious otherwise. Additionally, certain configurations facilitate the study of a dominant parameter, or the effect of certain parameters on different performance requirements. Certain mathematical considerations must be pointed out at this time concerning the interdependence of these graphical plots. Clearly, the construction of PARAM-7, the Bode plot, offers no problem. In this case, the desired values of alpha, beta, and omega are inserted into (2.5-3) and the corresponding magnitude squared value is then computed. For the remaining programs (PARAM-1) through (PARAM-6), a rearrangement of (2.5-3) is necessary since magnitude is no longer the unknown variable. As an illustration, consider (2.5-4) where alpha has been specified. Beta then becomes the unknown value to be solved. For added clarity it is better to revert back to (2.5-2) in order to point out why a second order system is always encountered in the linear case, as well as the non-linear alpha - beta product case. Since beta is the unknown variable, pre-selected values of magnitude, omega, and alpha must be presumed. Performing these substitutions, (2.5-2) may be written as

$$M^2 = \frac{[p_K + q_K \beta]^2 + [r_K + s_K \beta]^2}{[t_K + u_K \beta]^2 \quad [v_K + w_K \beta]^2}$$

where, using the notation of (2.5-4)

$$p_K = [b_0 A + e_0] - [b_2 A + e_2]\omega^2 + [b_4 A + e_4]\omega^4 - \cdots$$
$$q_K = [c_0 + d_0 A] - [c_2 + d_2 A]\omega^2 + [c_4 + d_4 A]\omega^4 - \cdots$$
$$r_K = [b_0 A + e_0] - [b_2 A + e_2]\omega^2 + [b_4 A + e_4]\omega^4 - \cdots$$
$$\vdots$$
$$w_K = [c_0 + d_0 A] - [c_2 + d_2 A]\omega^2 + [c_4 + d_4 A]\omega^4 - \cdots$$

Therefore, one can easily see that by squaring the right hand bracketed values and combining like terms,

$$M^2 = \frac{SNBO + SNB1\,\beta + SNB2\,\beta^2}{SDBO + SDB1\,\beta + SDB2\,\beta^2}$$

where the terms SNBO, SNBI, SNB2, SDBO, SDB1, and SDB2 are so chosen because they are the variables used in the computer program to be discussed later. Since the numerical value of M^2 has been specified, this equation can be cross-multiplied. Transposing the right hand side of the resultant equation, one can write:

$$SNB0 - M^2\,SDB0 + [SNB1 - M^2\,SDB1]\beta + [SNB2 - M^2\,SDB2]\beta^2 = 0$$

This is clearly a quadratic in beta, and employment of the quadratic equation yields the desired results. Imaginary numerical values have no physical significance for graphical plotting and are therefore discarded. However both the plus and minus signs in front of the square root term of the quadratic formula must be considered. Thus it is possible to have two values of beta for a specified combination of magnitude, omega, and alpha. This is an important consideration in graphical plotting techniques which can lead to spurious lines if not properly accounted for. Effectively the problem requires calculating all the prescribed graph points using first one sign of the square root term, and plotting them. Then the computational process is repeated with the alternate sign of the square root term. This illustration has shown how a solution would be obtained for beta when alpha, magnitude, and omega are known. Precisely the same procedure would be followed to obtain a solution for alpha when the values of beta, magnitude, and omega are given; only (2.5-5) would be used in lieu of (2.5-4)

RESONANT PEAK MAGNITUDE

Performance specifications of certain systems include stringent requirements on the presence of a stipulated resonant peak magnitude. Conversely, other design studies are concerned with the guaranteed absence of a resonant peak magnitude above a pre-determined level. Either of these requirements results in the study of frequency response curves with particular emphasis on the peak magnitude and the frequency at which it occurs. To facilitate such studies, it would be advantageous to construct resonant peak magnitude curves for the system as a function of the polynomial variable coefficients, alpha and beta. This involves the utilization of the non-parametric theory presented at the end of article 2.4.

Normally, one thinks of only one maximum point in the entire frequency spectrum because of dominant root characteristics of low order polynomials. Certain high order systems, however, can be represented by characteristic equations which have more than one pair of dominant complex roots. Such arrangements will yield more than one maximum point on the frequency response curve. Only one of these points can be considered as the global maximum, while all other peaks must be relegated to the role of local maxima. To avoid ambiguities, the notation $M_p(\omega)$ will be restricted to the global maximum point only. The corresponding value of omega will be denoted by ω_p. Since it is desirable to refer to local maximum points as well, the term $M_r(\omega)$ shall be construed to be applicable to <u>all</u> resonance points, both local and global. The respective angular frequencies at which these occur will be labelled as ω_r. Thus it is possible to have any number of $M_r(\omega)$ and ω_r values for a given transfer function. Only one of these can be the $M_p(\omega)$ and ω_p, however.

Recalling (2.5-3) where e_K and f_K are both functions of the variable parameters alpha and beta, it is possible to state this equation in the following functional notation:

$$M^2(\omega) = \frac{f_1(\alpha, \beta, \omega)}{f_2(\alpha, \beta, \omega)} \tag{2.5-6}$$

Assuming at least one pair of dominant complex conjugate roots, it is possible to find the following relation from (2.5-6):

$$M_r^{\,2}(\omega) = \left.\frac{f_1(\alpha,\beta,\omega)}{f_2(\alpha,\beta,\omega)}\right|_{\omega=\omega_r} = \frac{f_3(\alpha,\beta,\omega_r)}{f_4(\alpha,\beta,\omega_r)} \tag{2.5-7}$$

$$M_p^{\,2}(\omega) = \left.\frac{f_1(\alpha,\beta,\omega)}{f_2(\alpha,\beta,\omega)}\right|_{\omega=\omega_p} = \frac{f_5(\alpha,\beta,\omega_p)}{f_6(\alpha,\beta,\omega_p)} \tag{2.5-8}$$

Fundamental theory of equations dictates the following correlation between (2.5-6), (2.5-7) and (2.5-8):

1. A relation in the form of (2.5-7) exists whenever the slope of (2.5-6) is equal to zero and its radius of curvature is negative. Expressing this statement in mathematical terms:

$$\frac{\partial\left[M^2(\omega)\right]}{\partial\omega} = 0 \tag{2.5-9}$$

$$\frac{\partial^2\left[M^2(\omega)\right]}{\partial\omega^2} = 0 \tag{2.5-10}$$

2. Only that value of ω_r in (2.5-7) which globally maximizes (2.5-6) satisfies (2.5-8). Thus it is possible to state:

$$M_p^{\,2}(\omega) = \text{maximum}\left\{ M^2(\omega)\Big|_{\omega=\omega_r} \right\} \tag{2.5-11}$$

The preceding theory is adequate in that it gives the necessary and sufficient conditions for the existence of $M_p^{\,2}(\omega)$. Satisfying the above criteria is fairly straight forward whenever the values of alpha and beta have been selected or prescribed. The dilemma arises when seeking the algebraic relationship for $M_p^{\,2}(\omega)$ as a function of alpha or beta.

Although three equations are available, the inequality in the second derivative criterion of (2.5-10) imposes a severe limitation on the solution technique. The most straight forward approach would be to utilize (2.5-7). From these it is possible to obtain (2.5-11). Such a method of attack is only useful when both alpha and beta are given. This would require a two dimensional scan for all possible values of alpha and beta. Various computerized programs exist in optimal control theory for the determination of (2.5-7). These, however, are based on apriori knowledge or random search techniques using Lagrange multipliers. Not only are these methods time consuming, they are not conducive to highly sensitive frequency response curves where resonant peaks are in close proximity to one another. An alternative solution employs the non-parametric theory which results in (2.4-10). This procedure is applicable only when the two equations involved can be set equal to zero. Therefore it is necessary to investigate the locus of inflection points of the magnitude versus frequency curve. From this information, it is possible to determine how many local maximum points exist. It can be shown that for n inflection points, there can be no more than n maxima. In practice, however, only $n-1$ will be found for most systems. This procedure has the important restriction of eliminating omega. Thus it is impossible to employ (2.5-6) in order to find the value of $M^2(\omega)$ at each inflection point. At best, this result makes it possible to construct the locus of inflection points in the parameter plane when alpha and beta are the axis variables. It then becomes necessary to employ some other means of determining which side of this locus represents the minima, or maxima. Full utilization of the non-parametric theory can produce this desired result. Inspection of the developments leading to the coefficient matrix of (2.4-12) will show that the combined application of (2.4-10) and (2.4-12) provides the necessary information for (2.5-6) as follows:

1. The results of (2.4-10) yield a solution of the form:

$$X = f_7(\alpha, \beta) \tag{2.5-12}$$

2. The results of (2.4-12) provide the following alternate solutions:

$$Y_1 = f_8(\alpha, \omega) \tag{2.5-13}$$

or

$$Y_2 = f_9(\beta, \omega) \tag{2.5-14}$$

depending on which coefficient variable is eliminated. From these relations, it is possible to employ (2.5-6) directly by use of (2.5-12) and (2.5-13), or (2.5-12) and (2.5-14). In this manner, it would be possible to employ the optimal control theory previously mentioned. However the more direct approach would be the utilization of the gradient technique which is presently employed in many optimal control problems. Alternately, either of these pairs may be solved simultaneously to provide a locus of inflection points as a function of the two remaining variables, alpha and omega, or beta and omega. This latter approach would be most desirable for frequency response parameter plots employing these variables as the graphical axes.

The preceding theory has shown several ways of handling the peak resonant magnitude problem. Inherent in each of these, however, is a vast array of mathematical manipulations which is limited by solution techniques presently available. The following development will indicate this more clearly. Consider again the fundamental relationship of (2.5-3) upon which these derivations are based. The following result is obtained for (2.5-9):

$$\frac{\partial M^2(x)}{\partial x} = \frac{\left[\sum_{\ell=0}^{m} (-1)^\ell f_\ell x^\ell\right]\left[\sum_{K=0}^{n} (-1)^K e_K K x^{K-1}\right]}{\left[\sum_{\ell=0}^{m} (-1)^\ell f_\ell x^\ell\right]^2} - \frac{\left[\sum_{K=0}^{n} (-1)^K e_K x^K\right]\left[\sum_{\ell=0}^{m} (-1)^\ell f_\ell \ell x^{\ell-1}\right]}{\left[\sum_{\ell=0}^{m} (-1)^\ell f_\ell x^\ell\right]^2} \tag{2.5-15}$$

Let g_i = numerator coefficient of (2.5-15) with the sign embedded, and let h_ℓ = denominator coefficient. Thus (2.5-15) can be rewritten as:

$$\frac{\partial M^2(x)}{\partial x} = \frac{\sum_{i=0}^{m+n-1} g_i x^i}{\sum_{\ell=0}^{m} (-1)^\ell h_{2\ell} x^{2\ell}}$$

The equivalent form of (2.5-10) then becomes:

$$\frac{\partial^2 M^2(x)}{\partial x^2} = \frac{\left[\sum_{\ell=0}^{m} (-1)^\ell h_{2\ell} x^{2\ell}\right]\left[\sum_{i=0}^{m+n-1} g_i i x^{i-1}\right]}{\left[\sum_{\ell=0}^{m} (-1)^\ell h_{2\ell} x^{2\ell}\right]^2} - \frac{\left[\sum_{i=0}^{m+n-1} g_i x^i\right]\left[\sum_{\ell=0}^{m} (-1)^\ell h_{2\ell} 2\ell x^{2\ell-1}\right]}{\left[\sum_{\ell=0}^{m} (-1)^\ell h_{2\ell} x^{2\ell}\right]^2} \tag{2.5-16}$$

Thus from (2.5-16) it is noted that the order of the denominator polynomial is raised to a power of four times the order of the denominator in the original transfer function. In like manner, it is evident that the numerator polynomial of (2.5-16) is also of very high order. Therefore, all but the simplest of transfer functions would require computer assistance to solve the determinants of (2.5-15) and (2.5-16). Although many programs exist for explicit determinant solutions, these cannot be used when one of the terms is an unknown variable. Until a generalized computer program is developed to handle these cases, the construction of peak resonant curves can only ba achieved by a laborious calculation of secondary curves. Such procedures do exist, whereby tangent lines are drawn from a series of curves. To be highly accurate, however, an excessive number of these are required for sensitive systems.

PHASE CALCULATIONS

The preceding discussions in this article have been limited to magnitude studies in the frequency domain. An equally large field of interest is that of phase angle characteristics. The utility of the magnitude versus frequency plot lies not only in analyzing or predicting the frequency response of a system, but also in the ability to check its stability as well. The correlation between magnitude and phase curves provides this verification by means of phase margin and gain margin calculations. It therefore becomes desirable to derive the phase calculations as a function of the three variables: alpha, beta and omega.

Initial developments in this article showed how (2.5-1) could be altered to produce

$$T(j\omega) = \frac{N_{even}(j\omega) + N_{odd}(j\omega)}{D_{even}(j\omega) + D_{odd}(j\omega)}$$

Considering the numerator and denominator separately, either can be regarded as the real and imaginary components of a vector with a given magnitude and angle. Magnitude calculations have already been developed and are of no interest here. Phase angle calculations, however, are known to involve the arc-tangent of the vector components as follows:

$$\text{Phase Angle} \equiv \theta = \tan^{-1}\left[\frac{Im\{T(j\omega)\}}{Re\{T(j\omega)\}}\right] \tag{2.5-17}$$

Two solution procedures are therefore possible. One is to solve independently for the angle of the numerator vector and subtract the denominator vector angle from it. Thus:

$$\theta = \tan^{-1}\left[\frac{Nodd\,(j\omega)}{Neven\,(j\omega)}\right] - \tan^{-1}\left[\frac{Dodd\,(j\omega)}{Deven\,(j\omega)}\right] \tag{2.5-18}$$

The alternate approach involves only one trigonometric conversion. Clearing the imaginary term from the denominator of $T(j\omega)$,

$$T(j\omega) = \left[\frac{Neven(j\omega) + Nodd\,(j\omega)}{Deven(j\omega) + Dodd\,(j\omega)}\right]\left[\frac{Deven\,(j\omega) - Dodd\,(j\omega)}{Deven\,(j\omega) - Dodd\,(j\omega)}\right]$$

$$T(j\omega) = \left[\frac{Neven\,(j\omega)\,Deven(j\omega) - Nodd\,(j\omega)\,Dodd\,(j\omega)}{(Deven\,(j\omega))^2 - (Dodd\,(j\omega))^2}\right]$$

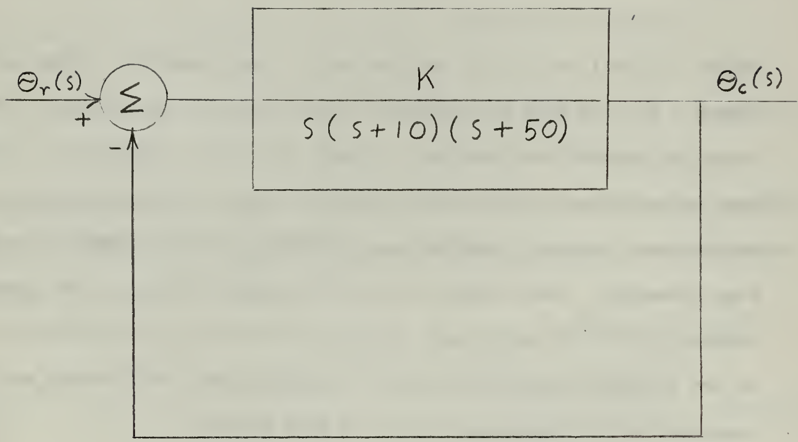
$$+ \left[\frac{Nodd\,(j\omega)\,Deven\,(j\omega) - Neven\,(j\omega)\,Dodd(j\omega)}{(Deven(j\omega))^2 - (Dodd\,(j\omega))^2}\right]$$

$$T(j\omega) = Re\{T(j\omega)\} + Im\{T(j\omega)\}$$

Therefore, substituting this into (2.5-17),

$$\Theta = \tan^{-1}\left[\frac{Nodd(jw)\,Deven(jw) - Neven(jw)\,Dodd(jw)}{Neven(jw)\,Deven(jw) - Nodd(jw)\,Dodd(jw)}\right] \quad (2.5\text{-}19)$$

Either (2.5-18) or (2.5-19) may be used for the prescribed phase calculation. For the Bode plot (PARAM-7), both have the same degree of difficulty in computer applications, although (2.5-18) is preferred on long hand calculations. Extensions to phase curves in the remaining parameter plane frequency response graphs (PRAAM-1 through PARAM-6) have not been attempted. These would be possible without the use of the quadratic formula if (2.5-18) were used. Selection of (2.5-19) requires the usage of the quadratic formula with both its square root coefficients as illustrated in the magnitude portion of this article.

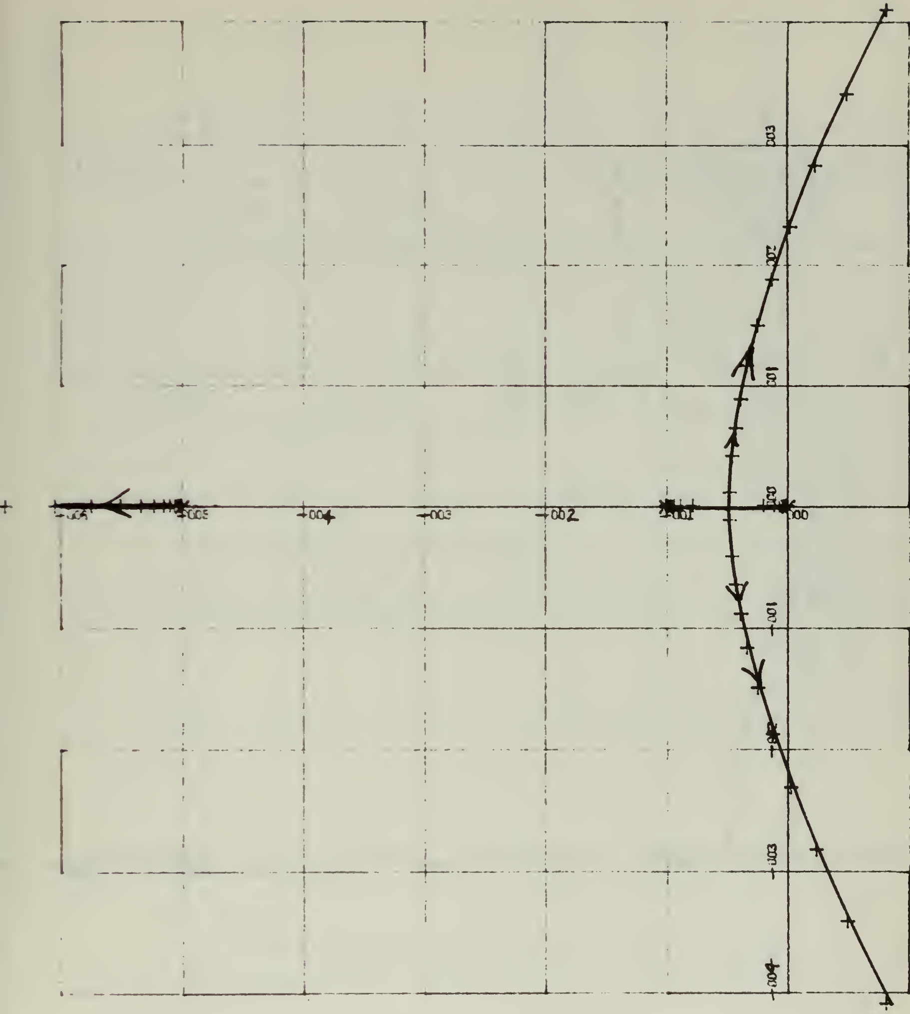


UNCOMPENSATED THIRD ORDER SYSTEM

CHARACTERISTIC EQUATION

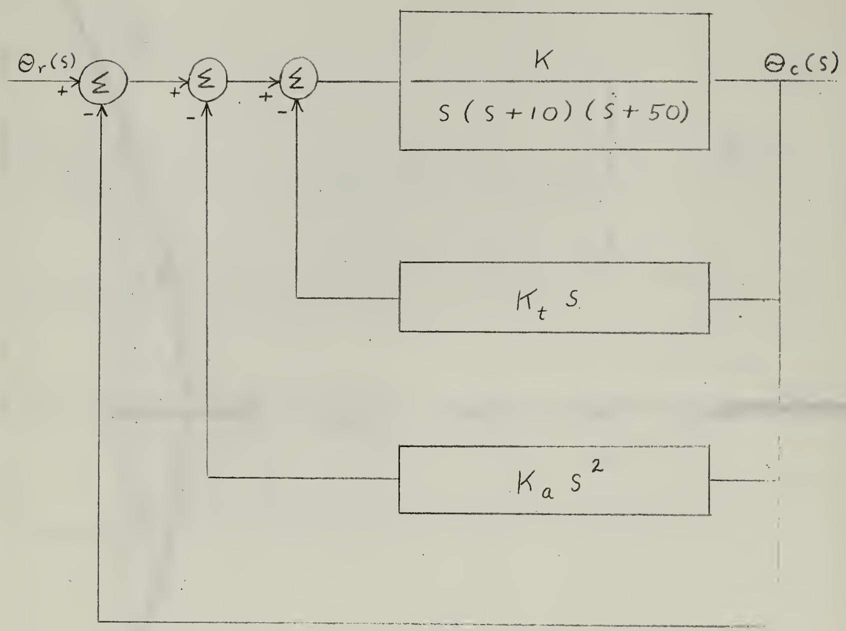
$$s^3 + 60s^2 + 500s + K = 0$$

FIGURE 2-1



X-SCALE=1.00E+01 UNITS INCH.
Y-SCALE=1.00E+01 UNITS INCH.
DAVIS, ROOT LOCUS FOR S**3+60S**2+500S+K=0
GAIN,(K) VARIES FROM 0 TO 100,000

FIGURE 2-2



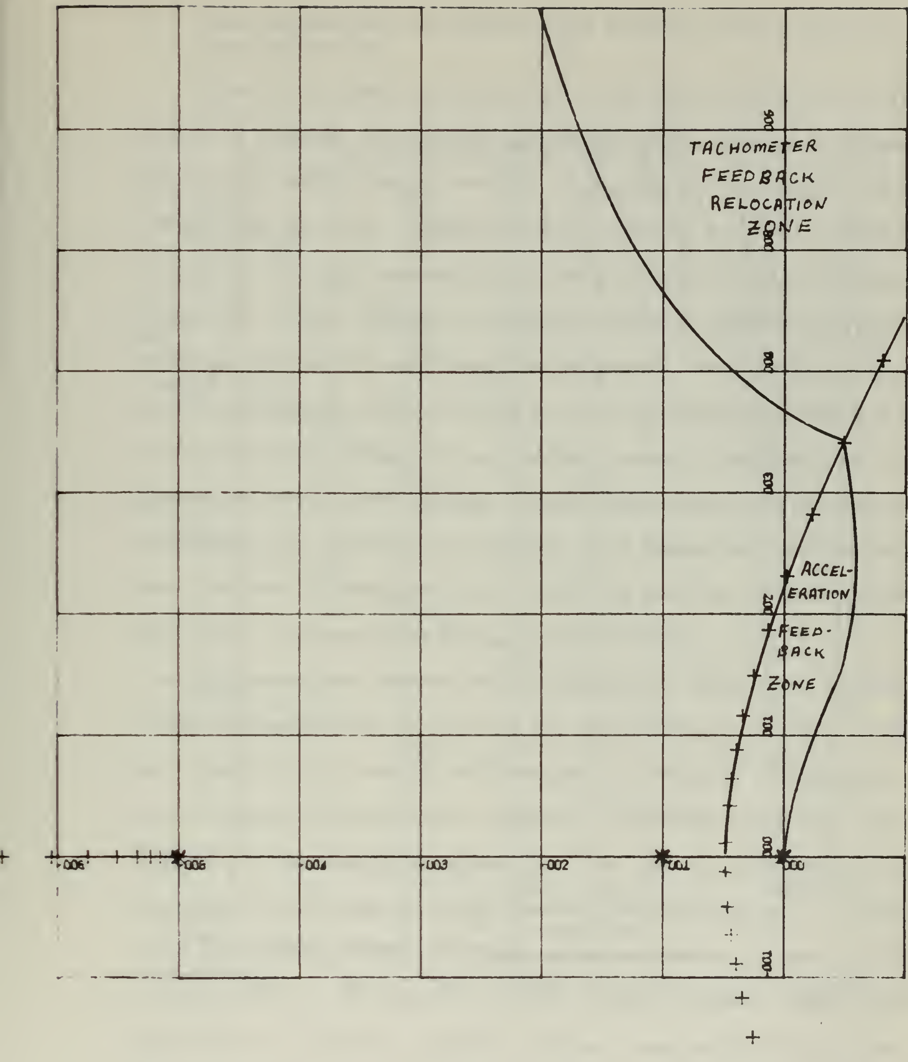
THIRD ORDER SYSTEM WITH TACHOMETER

AND ACCELERATION FEEDBACK

CHARACTERISTIC EQUATION

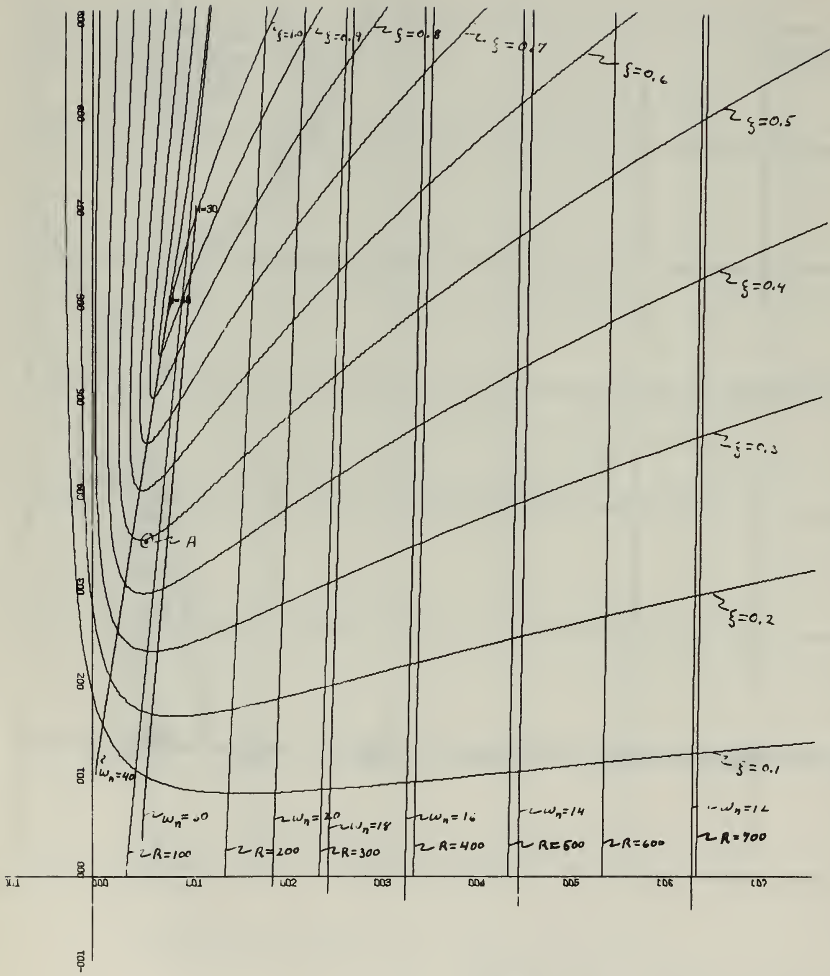
$$S^3 + (60 + KK_a)S^2 + (500 + KK_t)S + K = 0$$

FIGURE 2-3



X-SCALE=1.00E+01 UNITS INCH.
Y-SCALE=1.00E+01 UNITS INCH.
GLAVIS, ROOT LOCUS FOR S**3+60S**2+500S+K=0
FEEDBACK COMPENSATION ROOT RELOCATION ZONES

FIGURE 2-4



X-SCALE=1.00E-03 UNITS INCH.
Y-SCALE=1.00E-02 UNITS INCH.
GLAVIS, PARAMETER PLANE CURVES
S**3+(60+100,000A)S**2+(500+100,000B)S+100,000=0

FIGURE 2-5

3. Computerized Graphical Construction Techniques For Frequency Response Studies.

Graphical analysis has always held a virtuous distinction in all facets of industry and business. Since the Second World War, however, the advances of technology promoted the engineering profession beyond the existent graphical methods which were derived for the single purpose of varying only one parameter at a time. In spite of this, the ancient adage "one picture is worth a thousand words" has withstood the test of time, as witnessed by many practicing engineers today. Many pictorial representations of single variable systems are familiar; however with the advent of multiple parameter variations, graphical representation in two dimensions becomes more complex. Considerable experience and moderate imagination are necessary to visualize three dimensional configurations when presented in two dimensional planes, in order to adequately make an intelligent intrepretation for meaningful results.

The recent developments employing algebraic methods for variation of two parameters have resulted in the successful use of digital computer facilities to perform the required manipulations for graphical representation of mathematical equations. The preceding section was devoted to a fundamental development of the underlying mathematical theory employed in two parameter design studies. This section will include a broad presentation of certain graphical methods which are most likely to be employed in two parameter electrical control problems, with primary application to frequency response studies. This section is also concerned with a detailed inspection and description of the computer programs utilized in formulating those graphs. The discussion is not directed toward programming, but rather an effort is made to clarify certain computer oriented problems so that engineers who desire to utilize any program contained herein may do so with minimum difficulty and yet gain maximum benefit from its comprehensive construction. Illustrative examples of each of these programs appear in sections 4 and 5 where several case studies are investigated in depth.

3.1 Selection of Graphical Representations.

For any given set of six variables, the study of their respective inter-relationship by means of two dimensional projections results in a myriad of graphical portrayals. The magnitude of the problem can best be seen in the following development: Assume that an arbitrary number of variable parameters are to be considered. If n is the number of design variables, it is possible to show that the number of possible graphical representations is n-factorial ($n!$). All graphs reveal essentially the same information content when one axis variable is switched from the ordinate to the abcissa, and vica versa. Therefore, redundancy is eliminated by allowing the abcissa and ordinate variables to be interchangeable. This reduces the number of combinations by a factor of two, thereby making the revised number of graphical representations equal to ½ ($n!$). Although a considerable reduction in the size of the array has been effected by interchanging the axis variables, it is highly desirable to reduce this size further, since a six variable array means 360 graphs, a five variable array requires 60 representations, and so forth. To further reduce this compilation, it is often possible to interchange any two parameters alpha (α) and beta (β). Such would be the case in the parameter plane technique where they are, by definition, arbitrary variables of the characteristic equation. Even this reduction leaves a formidable task of evaluating over 180 graphs for a system incorporating six variables, and any system with five variables requires an array of 33 different graphical representations. For this reason alone, the present state of the art is limited to at most four variables for practical design techniques employing graphical displays. Any system with four variables requires basically seven different graphical presentations for comprehensive evaluation. Seldom are all of these required for a satisfactory design, although barring the elements of time and expense they would most likely be necessary for optimal results. Effectively the problem at hand involves the inspection of four independent variables. This can be regarded as a four dimensional space which is not physically realizable. By assigning a constant value to one of these four variables, the problem is reduced to a three dimensional sub-space representation which does have a physical shape. It's meaning, however is restricted to that particular value of the fourth variable. To present this three dimensional sub-space on a two dimensional plane requires that a large number of curves be projected onto this plane, where each curve represents a different value of the third variable. The following discussion is based on this concept, and the resultant graphs should be interpreted accordingly.

Most specifications for the dynamic performance of linear systems are written in terms of the time domain and usually include such terms as peak overshoot, transient duration, and steady state error. It therefore becomes necessary for the control engineer to bridge the time domain with the more utilitarian frequency domain. Correlation is accomplished by means of the Fourier integral when rigorous methods are called for. This treatment is avoided in practice however, because of the inherent arduous task involved in all but the simplest of integrations. It can be shown that by suitable approximation, many linear problems fall into the category of second order systems. For engineers not experienced in partitioning methods and root locus techniques, this statement would seem to be easily contested; however, such an analogy actually exists in most cases. For systems which cannot be resolved into an equivalent second order approximation, the absence of dominant complex roots is evidenced by the shape of the closed loop frequency response curve. Since considerable information exists in present day literature on the generalized second order system, much time and labor is saved by manipulating a given problem into this form whenever possible, and extracting the equivalent damping and bandwidth specifications from pre-computed curves and tables.

A cursory review of the terms used in control engineering applications will reveal that some of the more common design criteria are steady state magnitude (M), phase angle designated theta (θ), natural angular frequency designated omega (ω_n), and damping ratio designated zeta (ζ). Some problems are more amenable to the rectangular coordinates of the complex frequency plane. These are designated as the damping constant sigma (σ) and the damped angular frequency omega (ω). It is again emphasized that two forms of angular frequency are designated by the word omega. Much of the existing literature uses this term very loosely, and extreme care must be exercised not to mistake undamped, or natural angular frequency (ω_n) for damped angular frequency (ω). The relation between these two variables is found in article 2.1, and it should be noted that these terms are synonomous only when the damping ratio (ζ) equals zero.

Section 2 developed the notational concept of alpha (α) and beta (β) as representing two arbitrary variables embedded within the coefficients of the characteristic equation. The preceding paragraph has shown that eight variables are frequently encountered in the resultant electrical engineering control problems. As previously stressed, the simultaneous usage of all of these is not feasible. Fortunately, however, one may break the control problem down into two basic categories: stability and frequency response. These two facets are interrelated, however, and for that reason some attention must be given to system stability when performing frequency response studies.

The parameter plane stability problem may effectively be considered as the study of the relation between the characteristic equation polynomial and the stability performance variables ζ and ω_n, or σ and ω. When the two parameters alpha and beta are embedded within the characteristic equation, the scope of the problem increases to a study of at least four variables. Thus one might be inclined to consider seven possible graphical arrangements. Unlike the frequency response studies, however, the stability problem has been attacked from a slightly different viewpoint. Only alpha and beta have been plotted along the axes, and no attempt has been made to develop the necessary equations and programs to portray a different set of graph coordinates. Thus only one basic graphical presentation is used for parameter plane stability studies. The example in section 2 illustrated this method of stability study by employing the following alpha - beta parameter plane curves: constant zeta (ζ) curves; constant omega (ω_n) curves; constant zeta - omega (ζ-ω_n) curves; constant sigma (σ) curves; constant omega (ω) curves; and real root ($\sigma + j0$) curves. Although the constant zeta - omega (ζ-ω_n) curves are identical with the constant sigma (σ) curves, the algebraic methods required for each are different. The names assigned to each have therefore been kept separate to avoid confusion in the respective graphical construction techniques. Additionally, the real root ($\sigma + j0$) curves represent constant sigma points in the complex frequency plane which are transformed into lines on the parameter plane. As such, they are not to be confused with the constant sigma (σ) or constant zeta - omega ($\zeta - \omega_n$) curves. Although partial programs exist for these curves, a program entitled PARAM-D has been written for the primary purpose of consolidating these programs into one package for added flexability and versatility. This permits any combination of the above curves to be processed on one graph.

Complex engineering problems often involve a stability study of systems with more than two arbitrary characteristic equation coefficients. Program PARAM-D has been extended to include a third variable which is herein defined as gamma (γ). This scheme will allow stability analysis studies to be visualized in three dimensions, rather than two. When confronted with this type of problem, it then becomes essentially a four variable problem. Therefore it is necessary to hold one of these four independent variables constant, and generate a series of curves using the remaining three. This is accomplished by assigning a constant value to one of the stability performance variables: ζ, ω_n, $\zeta - \omega_n$, σ, ω, or $\sigma + j0$. The remaining three variables are those in the characteristic equation coefficients, namely alpha, beta and gamma. With these, the program attemps to develop a three dimensional surface for the fixed value of the specified stability performance variable. The x, y and z axis variables are alpha, beta and gamma respectively. Graphical presentations are restricted to two dimensions, however. The dilemma therefore is that this three dimensional surface must be projected onto a two dimensional plane, which is herein defined by the x and y axes. The only way that this projection can be accomplished is to construct a large number of third variable (gamma) curves on the graph. Each curve then represents a cross section of the three dimensional surface. The physical shape of the surface can then be visualized providing a sufficiently large number of these curves are thus constructed.

The frequency response study has been viewed in a manner which is slightly different than the above stability program approach. It was initially decided to make a thorough analysis of the four variables: α, β, M, and ω. As such, alpha and beta can be regarded as two variables embedded in the characteristic equation coefficients, while magnitude and omega are the two frequency response performance criteria of interest. Treating these as four independent variables results in seven basic graphical combinations. These have been titled PARAM-1 through PARAM-7. Additionally, phase is investigated in PARAM-7, where its correlation with magnitude is most easily made. All possible graphical presentations of interest have been utilized, and a summary of these is presented in figure 3-1. They have been implemented in one consolidated computer program PARAMS which is included in appendix I. A brief summary of these graph characteristics is presented below, while a more thorough discussion is reserved for article 3.2.

PARAM-1 is a graph of alpha versus beta, with magnitude held at a fixed value while drawing constant omega curves. Interchanging alpha and beta produces absolutely no alteration of curves, other than a ninety degree re-orientation. PARAM-2 is a graph of alpha versus beta again, with omega held at a fixed value this time, while constant magnitude curves are drawn. As in PARAM-1, no difference other than a ninety degree re-orientation is obtained when the axis variables are interchanged. PARAM-3 places alpha on the ordinate, and magnitude on the abcissa. Omega is held at a fixed value throughout the computational process, while constant beta curves are generated. Now interchanging alpha and beta in PARAM-3 will result in an entirely different set of curves, since the sensitivity of these two variables is normally different. PARAM-4 is a graph of alpha on the ordinate versus omega on the abcissa. Beta is held constant while magnitude curves are generated. Again, different curves will be obtained when alpha and beta are interchanged, since holding one characteristic equation parameter constant will produce an entirely different effect if another parameter is held at a fixed value instead. PARAM-5 generates constant omega curves with beta held at a fixed value, and graphs these curves against magnitude on the abcissa, and alpha on the ordinate. As with PARAM-4, the PARAM-5 curves are quite different when the roles of alpha and beta are interchanged. PARAM-6 is a graph of alpha on the ordinate and omega on the abcissa. Magnitude is held at a fixed value throughout the calculations while constant beta curves are generated on the alpha-omega plane. This is the same arrangement as PARAM-3 with the roles of magnitude and omega being reversed. Interchanging the alpha and the beta in PARAM-6 will also cause a sharp alteration in the curve configurations. The familiar Bode plot is herein reproduced as PARAM-7. Needing no review, it is simply a logarithmic plot of magnitude on the ordinate, and omega along the abcissa. With beta held fixed at some constant value, constant alpha curves are generated. Conversely, when alpha is held fixed at some constant value, the beta curves are generated and plotted on the magnitude - omega plane. One problem exists with the logarithmic plot, however. In order to draw a useful logarithmic grid, it was necessary to shift the origin to the lower right hand corner of the graph paper. In this manner, the decade span could cover a total of 15 inches rather than a maximum of nine. The net effect has been to require the abcissa to be plotted along the right hand edge of the graph while the ordinate is plotted along the bottom where the abcissa otherwise would be, thereby coinciding with the X-axis. Thus the ordinate originates in the bottom right hand corner and increases toward the lower left hand corner, or original origin. The same process is required for PARAM-4 and PARAM 6 whenever specifying the logarithmic plots. Detailed coverage of the available options will be found in article 3.2 which is devoted to program PARAMS.

The programs PARAM-D and PARAMS utilize only four variables, although their adaptation to others has been made. The method of solution, however, has been with two variables along the axes, while the remaining two have been employed as constant curves, and in the case of program PARAMS one variable has been ehld fixed at a constant value. In program PARAM-D, the axis variables have been selected as alpha and beta only; no attempt has been made to develop any other projection whereby other variables might replace alpha and/or beta as axis coordinates. Program PARAMS has attempted to focus attention on several two dimensional projections in the hopes that peculiarities not evident in one planar presentation might become conspicuous in another mode. Due to the extensive additional projections with the incorporation of one more variable, it was decided to refrain from including phase (Θ) as a parameter in all but the Bode diagram, PARAM-7, where its utility and value have already been demonstrated.

The following article will develop the basic capabilities of program PARAMS with regards to its application in frequency response studies.

3.2 Digital Computer Program PARAMS.

The preceding article and earlier example in section 2 presented in some detail the capabilities of program PARAM-D for stability studies. Program PARAMS has been developed to study some pertinent factors in frequency response which might prove useful in systems and controls design.

For each of the graphical representations entitled PARAM-1 through PARAM-7, a separate computer program was developed in FORTRAN II language for use on the Computer Data Corporation (C.D.C.) Model 1604 digital computer [11]. Each of these programs was compiled in a similar fashion, and employed the algebraic methods derived by Doctors G. J. Thaler and A. G. Thompson [12]. As previously discussed in this section, each graphical configuration has been developed in order that the controls design engineer might gain additional tools to facilitate studies in analysis and synthesis. For the study of one particular problem, it is often necessary to analyze the systems behavior from many different angles, and then select a smaller number of characteristics for further intensive study. Unfortunately, every real life problem involves a different set of constraints, and for that reason there can be no set pattern for successful design application in all cases. Many situations reduce to trial and error techniques on preliminary studies in order to obtain the best compromise when a number of possible solutions exist. The ability to accomplish this objective using many separate computer programs is indeed possible but cumbersome. Instead of this method, one consolidated computer program titled PARAMS has been written in FORTRAN IV language for use on the International Business Machine (IBM) model 360 computer. The entire computer program, together with explanatory flowgraphs and illustrative printouts, has been included as Appendix I. Details of its use, together with the advantages and disadvantages, will now be presented.

Versatility is indeed a virtue, provided it does not gain domination over simplicity. While considerable apprehension might be the first reaction upon reviewing Appendix I, a systemized pattern will become evident upon further inspection. With this program, complexity does not enter into the computational procedures, but rather a considerable amount of detail has been included in the printout format for the sake of clarity when reviewing the material contained within the graphs. At most, the computational process is intricate in the initializing stage wherein the various axis maximum and minimum limits, together with the curve and fixed constant values are redesignated for the ensuing common computational and graphical construction sections. This can be verified by noting that the numerator and denominator coefficient calculation sections are common to PARAM-1 through PARAM-6. It will further be seen that the gain computation for PARAM-7 employs different formulae but precisely the same logic, and for that reason is incorporated in the same numerator and denominator coefficient calculation sections as used for PARAM-1 through PARAM-6. Only the phase calculations for PARAM-7 require a different process for generation of the coefficients. For this reason, this computational section is bypassed for all other processes.

The logical commencement point for design studies is to define a range of parameter values which guarantees all roots to lie in the left half plane, for clearly the question of stability must remain foremost in the designer's mind. Program PARAM-D has been devised to meet this demand, and its capabilities to provide a stability perspective have already been delineated. In determining the various possible frequency response characteristics for a given system, several graphical representations will be invariably studied. Most likely, however, program PARAM-7 will be employed in the final analysis as an overall check on system specification criteria. The remaining programs, PARAM-1 through PARAM-6 are most informative in the initial and intermediate stages of design study. It would therefore seem that the inclusion of all frequency response parameter plane programs into one package would be most useful. Different design studies will normally emphasize certain performance criteria more than others. Thus seldom would the same sequence of graphical presentations be called for in two consecutive studies. In fact, PARAM-7 might be omitted for the initial coarse adjustments, but yet included in the final stages of refined improvements. In this manner each system study can be tailored to meet certain specifications and suit specific needs. Also, certain systems will be more complex than others. Therefore, more than one mathematical model might be considered for simultaneous inspection. The program is constructed to handle any number of such studies at one time. Each of these normally requires a complete set of data cards, and a brief explanation of their usage will follow. Additionally, an abbreviated description section appears in the program preface, found in Appendix I. The total number of these data card sets must be indicated on the first card following the main program. This desired number of runs is designated by the variable NRUN. The program simply processes the remaining data cards by commencing a "DO LOOP" with the variable IRUN going from 1 to NRUN. The first card of each data set indicates which graphical presentations (PARAM-1 through PARAM-7) are desired. Again, there is no limit to the number of these. The program simply commences another "DO LOOP" at statement number 26, with IPARAM going from 1 to KPARAM. This cycles each of the graphical constructions in the order called for. In this manner, a great multiplicity of variations exist for the simultaneous study of different preoblems. At this point it is pertinent to indicate a technique by which additional finesse may be utilized. Although most studies involve extensive rearrangement of a data set from one system to another, some require only a slight modification. In these latter cases, it is not necessary to submit an entire additional set of cards for each study. By first specifying the desired graphical plots of one system, it is possible to consecutively list the desired sequence for another system. Then a simple insertion of the desired data modification immediately following statement number 26 will alter the original data in the specified manner.

As an example, suppose one desired to first construct PARAM-5 with alpha on the ordinate, and then with beta on the ordinate. Simply list KPARAM as 2, and then indicate 5 twice for two consecutive plots of PARAM-5. Insert the following statement on an additional card behind statement 26: "IF (IPARAM. GE. 2) MODORD = 2". Thus the required change in the input data is effected at the desired time by one card rather than a complete set of data cards.

Although no standard names have been assigned to these different programs the title of each graphical representation (PARAM-1 through PARAM-7) has been retained to eliminate confusion in terminology for those persons previously exposed to the partial programs which are already in existence. The result has been a rather complex programming sequence for printout of graphical information and preliminary initializing computations, but as has already been stated, the basic manipulations are common to all programs, and are centrally located in the program from statement number 350 through statement 426. A cursory inspection of these 76 statement numbers as well as the remainder of the program will reveal that its construction is modular in form, and each section is a separate entity unto itself with adequate illustrative sub-titles. The overall program has been so arranged that many sections can be removed if it is desired to employ their use elsewhere. Conversely, additional extensions are easily facilitated by this manner of program fabrication.

Careful consideration has been used in this program design to retain flexibility for use on multiple problem types. Although the program is slightly larger in size than any of its component programs which were previously mentioned, it is smaller than the combination of any two of them. While intensive effort to streamline programs is not of particular significance, the repetition of computer processing is needless and inefficient. Also, the preparation of data cards for a large number of programs serves only to increase the probability of error with consequential waste in time, expense and labor. More important is the fact that computer programming is still a dynamic field which often necessitates program modifications to meet the requirements of more sophisticated equipment. Thus one might be motivated to use a single computer program in lieu of eight separate ones, and to this extent consolidation is warranted.

To ensure adequate clarity of results, an elaborate printout of information has been established. This information can be basically catagorized into three groups, each of which is controlled by a single input data variable IWRITE. The first reproduces all input data for verification, and is effected whenever the value of IWRITE is any non-negative number. The second category gives detailed information about how each graph will be constructed. Included in this section are the graph titles, the range of values for the ordinate and abcissa variables, the curves to be generated, and the parameter that is to be held constant throughout the computational process of that particular program. This explanatory section will be printed for all positive values of IWRITE. Finally, the computed results for each curve are printed out at pre-selected intervals. The value assigned to IWRITE determines at what intervals the computational results will be printed, and the range of permissible values is from one to 900. Thus an assigned value of one would result in the reproduction of every calculated curve value, whereas a value of ten would give a printed value for every tenth calculated point. Approximately one third of the program is devoted to printing out the above information for feedback purposes. If desired, these sections can be partially or entirely bypassed by the appropriate selection of the input data variable IWRITE. Any negative value assigned to this variable will provide complete calculation, but without any printout whatever.

It has already been pointed out that if the roles of alpha and beta are allowed to be interchanged, a much smaller number of graphs are required. In order to accomplish this objective, some criterion must exist for the choice of which variable (alpha or beta) will be plotted on the ordinate in the case of PARAM-1 through PARAM-6, and which variable will be held at a fixed constant value while the other generates the desired curves in the case of PARAM-7. The input data variable titled MODORD has been designated for this purpose, and so by this selection the type of curves is controlled. With a value of one for MODORD, alpha curves will be generated in the case of PARAM-7, and alpha will be plotted on the ordinate for the rest of the programs. Conversely beta will be used in lieu of alpha for those functions when the value of MODORD is set equal to two. This flexibility for PARAM-1 and PARAM-2 might be unnecessary, because as indicated earlier, a ninety degree reorientation of the graph is the only effect of interchanging alpha and beta when these variables are on the axes. However, the present day graphical plotting facilities are generally constructed in such a manner that the abcissa, or X-axis, is more severely restricted in length than the ordinate, or Y-axis. The facilities available at the inception of program PARAMS were in fact such that the ordinate length could be approximately twice as great as the length of the abcissa. It was envisioned that in some cases an expansion of one variable might be more desirable than another. Extending the applicability of MODORD to PARAM-1 and PARAM-2 makes this provision possible, which otherwise would require modification of the main program to achieve such results. Furthermore, it is advantageous to retain a common orientation of graphical construction when viewing several projections simultaneously.

The value of magnitude often occurs in several different forms, and thus it provides a source of confusion if the terminology is not clearly specified. The input data variable IMAG serves to eliminate this potential error by distinctly categorizing magnitude into one of three forms. The program is constructed in such a manner that the form of magnitude as specified by IMAG is maintained consistently for all graph plots. Although this is strictly a matter of convenience, it was found that various approaches to design require magnitude to be handled in different forms; hence this inclusion. Thus for magnitude to be interpreted as decibels, a value of zero would be assigned to IMAG, and the curve values of PARAM-2 and PARAM-4 would be listed in decibels, while the abcissa values of PARAM-3 and PARAM-5 would likewise be interpreted accordingly. If actual magnitude values were to be used instead, IMAG would necessarily have to be changed to one. Assigning a value of two to IMAG would dictate all values to be handled as magnitude squared. In all facets of life it seems that most conventions are fraught with exceptions, and this program has not escaped that dilemma. Since all Bode plots associate magnitude with decibels, it was decided to maintain this practice; consequently the selection of IMAG has no effect on the form of PARAM-7 graphs. They automatically plot magnitude along the ordinate in the form of decibels. Conventional practice has also dictated that omega be logarithmically plotted along the abcissa for all Bode diagrams. Other frequency response graphs which also plot omega on one axis have not acquired any standardized form, and for that reason provisions have been made to select a square linear grid or a logarithmic grid as deemed appropriate. The input data variable IGRAPH provides this option for the various possible combinations of PARAM-4, PARAM-6 and PARAM-7. Additionally, this variable is used to prevent any graphs from being generated by assigning to it a value of minus two. This provision is useful only for efficient use of available hardware when desired graph scales are initially unknown.

In order that maximum versatility might be gained, the memory storage requirements of this program have been expanded to present day capabilities. A review of the program DIMENSION statement just preceding the input data read in section will show that the bulk of the storage capacity is consumed by the two dimensional variables S through W. These are employed for the various optional arrangements of PARAM-7 and may be combined into one common storage array for all other calculations when employing this program in smaller computer facilities. In this manner, the memory core may be reduced from 500K bytes to less than 200K bytes. This is accomplished by inserting the following statement after the dimension statement: EQUIVALANCE (S,T,U,V,W).

The additional variable, phase, has been incorporated in PARAM-7 with several variations possible. The input data variable IPHASE is used to govern the relation between the magnitude and phase plots. No phase curves will be drawn when IPHASE is set equal to a negative number. Setting IPHASE equal to zero will enable phase curves to be superimposed upon the magnitude curves for ease in investigating stability by means of phase and gain margins. Assigning any positive number to IPHASE will provide separate phase curves. To allow for various superpositions, and various separate phase curve configurations as well, it was decided to employ an additional variable IQUAD which allowed the phase graphs to be plotted in the following two manners: When IQUAD has an assigned value of zero, phase curves will be automatically scaled at 45 degrees per inch, with 180 degrees on the magnitude axis. When the variable IQUAD is positive, however, the phase curves will be plotted with a scale equal to the value of IQUAD, starting with 0 degrees at the top of the graph. In this manner, it is possible to cover any desired range of phase angles.

Basic conformal mapping theory indicates that a polar plot can be generated from the information contained in the magnitude and phase plots. Particular importance might be attached to the polar plot representation of a system when the Nyquist criterion is preferred over the Bode stability analysis method. Although it is a duplicative process and therefore not strictly required for information content, its inclusion has been assumed appropriate for the sake of completeness. The basic construction of the polar plane and polar curves are contained within the program statement numbers 537 and 454 under the heading "PARAM-7 Polar Plot Curves". Its usage is governed by the rather appropriate input data variable IPOLAR, and a polar plot will be drawn when its value is one. Conversely, no polar plot will be drawn for any other value of IPOLAR.

An additional provision has been incorporated in program PARAMS for extended usage of the Bode plot. Certain advanced feedback design techniques call for complex systems to be studied from a cumulative standpoint. Such design measures require vector addition of component Bode diagrams and as such, the linear addition of each magnitude plot and each phase plot will yield erroneous results. Instead, the vector quantities of each design segment are summed in complex notation, and the final result is then plotted as a single magnitude curve, and also a single phase curve when the latter is requested. The program subsection designated to accomplish this process is titled "PARAM - 7 Superposition Section", and is contained by the program statement numbers 520 and 529. It is controlled by the input data variable NPAR7, where the magnitude of this variable dictates how many component terms are desired to be added together. It is necessary to emphasize at this point that this section performs a vector superposition of feedback terms, and not merely a scalar superposition. Thus whenever NPAR7 is greater than one, this section will add NPAR7 minus one terms vectorially to the first term before graphing the results. In order that these components may be properly calculated, provisions have been made to insert the component equation terms in the same coefficient format as normally submitted for the conventional calculations. These additional terms, however, are appended to the initial group of input data cards. Therefore NPAR7 minus one groups of coefficients must be submitted directly behind the title cards of the rest of the program.

A brief summary of the basic operational procedures which are common to each of the programs will now be undertaken. The schematic corollary in appendix I will parallel the following outline and should be consulted together with the main program for more detail.

The first six statements in the "Input Data Read in Section" provide, in proper FORTRAN language, the necessary format for most of the read and write statements of the program. Their use is specified in the "Data Card Information Section" which appears at the beginning of the program. The remainder of the input data read in section is devoted to reading all the 19, 21, or 23 data cards as outlined in the preceding information section. The next step is to print out the input data when so requested for verification. This procedure is accomplished by the section bounded by the statement numbers seven through 23, inclusive. Following the input data verification section, the program initializes the various parameters and computes the necessary controlling variables from such input data as axes maximum and minimum values. Within this section, contained by statements 24 to 54, the desired value of NPARAM is selected and this value guides the rest of the program sequence by assigning the proper axis variables and their limits, the correct number and type of curves to be generated etc. When the logarithmic plot is requested for PARAM - 4 or PARAM - 6, and whenever the Bode plot is to be generated, the section titled "Log Grid Graphical Construction" is executed. Extending from statement number 55 to 58, this sub-section utilizes the information generated in the initializing section to determine the ordinate scale and origin shift if applicable. Then it calculates the required number of decades by rounding up the maximum limit of omega to the nearest power of ten, and rounding down the minimum omega value to its nearest power of ten. Utilizing these factors and the axes lengths, a logarithmic grid is constructed in such a manner that the omega values are properly labelled along the right hand edge of the graph paper, while the ordinate values are indicated across the bottom in such a manner that their magnitudes are correct but the signs reversed. Thus the ordinate scale must be corrected to read increasing from right to left. Although this arrangement is less than optimum, it has proved to be much better than some alternatives which were previously attempted.

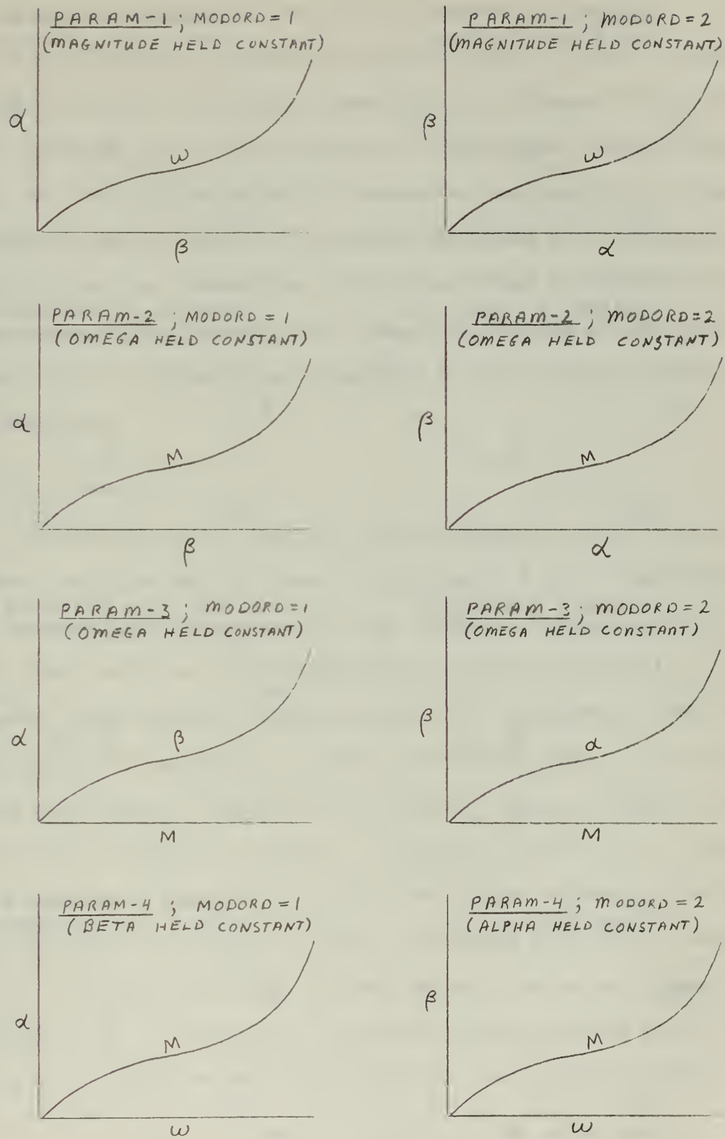
The most prolific section of the program ranges from statement number 59 to 200 under the sub heading "Graphical Information Printout Section". Its utility lies in the desire to establish positive correlation between printed data and graphical presentations when several similar graphical analyses are computed simultaneously. Following the extensive information printout, the main computational process commences by initiating calculations for the first curve with statement number 202. Next, the coefficient preceding the quadratic formula square root is assigned by the variable ZSIGN. Following this designation, the abcissa is incremented 900 times from the minimum specified value to the maximum value. Additionally the curve value is selected, based upon the value of the variable in statement number 202. Then the appropriate parameter which is to be held at a fixed constant value is assigned its pre-selected value. The ensuing calculations commencing with statement number 350 are in conformance with the theory presented in section 2. The magnitude protion of the Bode program requires the coefficients designated SUMN for the numerator and SUMD for the denominator, while the remaining programs utilize the numerator variables SNB2, SNB1, and SNB0 in addition to the denominator variables SDB2, SDB1, and SDB0. Such requirements are generated under the sub headings "Numerator Coefficients Calculations" and "Denominator Coefficient Calculations". Following these sections, the necessary phase coefficients are computed from statement number 386 through 400, and the additional Bode curve ordinate value calculations are continued through statement number 410. The section titled "Quadratic Formula Solution of Dependent Variable, PARAM-1 through PARAM-6" is self explanatory and simply computes the ordinate value commensurate with the assigned fixed value of the parameter which is kept constant, together with the specified value of the curve which is being generated, and the corresponding point on the abcissa. The computed results are then printed in accordance with the provisions contained between statement number 426 and 448. Having computed this ordinate value, the abcissa which is the independent variable is again incremented and a corresponding ordinate, or dependent variable point is calculated. After reaching the maximum permissible abcissa value, the process is again repeated for the alternate solution of the quadratic formula. When both possible ranges of values have been scanned, the next curve is specified, and the entire procedure repeated. Upon terminating the curve requirements, the program then reverts to statement number 26 for the next cycle in the run. When all requested graphical construction cycles have been accomplished, the next run is processed in like manner. The overall program is terminated when all desired runs have been computed and graphed.

The graphical plotting procedures will be discussed separately because their intricate operations tend to destroy the continuity of the preceding discussion on the overall program operation. Basically, the graphical plotting subroutine is executed whenever the following conditions are engaged: 1) upon initially setting up the logarithmic plot when specified in accordance with the above dictates; 2) when two or more computed points fall within the prescribed graph range and the next point lies outside the ordinate graph limits; 3) when the end of each scan of the abcissa has been reached - once for each value of the quadratic equation square root coefficient; 4) at the end of each set of curves for one graph, as accomplished by the graphical termination section found at the end of the program beginning with statement number 999.

The versatile arrangement necessitates several provisions for the many possible combinations of the Bode plot. Since the quadratic formula is not employed in these calculations, only one incrementing scan of the abcissa is performed. Additionally, the computed magnitude values have been programmed in such a manner that when the ordinate graph limits are exceeded, the external points are fictitiously redefined to lie on the appropriate ordinate graph limits. This procedure was necessitated by the additional phase calculations which remained within the graph limits throughout the entire range of the prescribed omega values. Finally, a separate provision had to be made for the construction of the phase curves, since an additional logarithmic graph was required when phase curves were plotted separately. This procedure has been accordingly programmed in the section titled "PARAM - 7 Phase Curves" found between statement numbers 529 and 537. The section titled "PARAM - 7 Polar Plot Curves" has already been discussed and is a separate entity unto itself, utilizing the pre-computed values of magnitude and phase, and plotting same in polar coordinates. This process is accomplished between statement numbers 537 and 999 in the program.

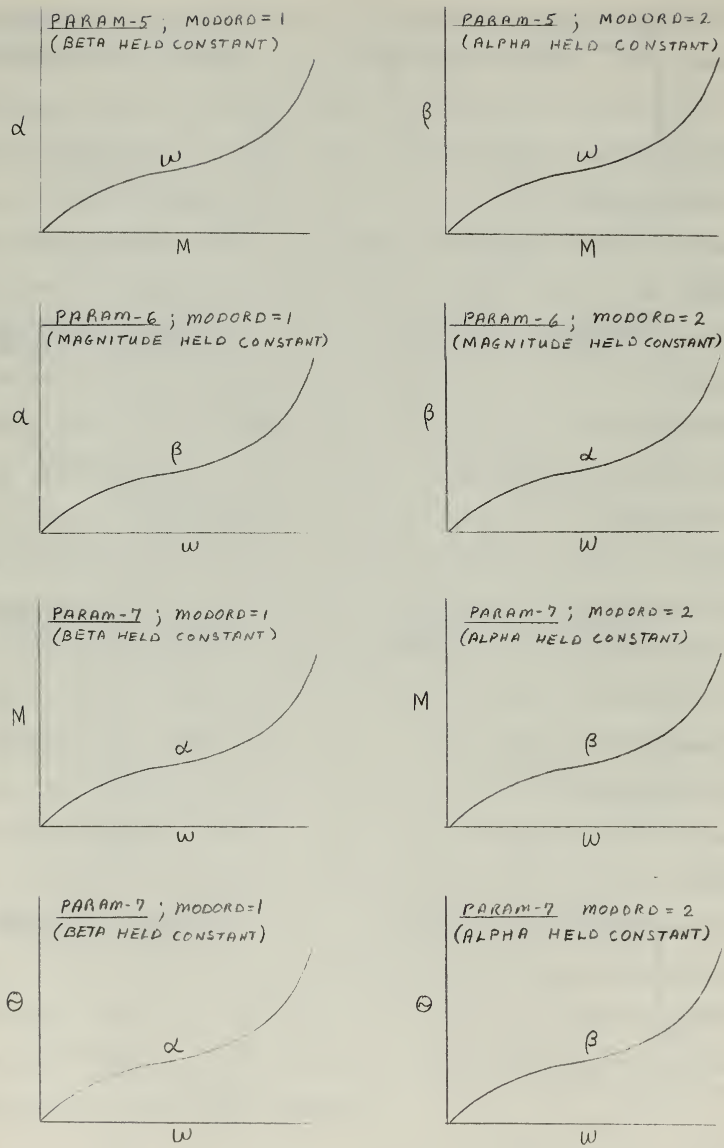
The preceding development has summarily shown that this program contains broad applications, and is well suited for additional modifications to meet particular needs. Blanket provisions for another graphical configuration has been purposely included in the program sequence labelled NPARAM- 8, and should future expansion be deemed appropriate, the required modification will merely entail filling the appropriate spaces with the desired initializing variables and applicable computational equations. To facilitate such extensions and also to enable maximum comprehension of the program operations, two logic flow diagrams have been included in Appendix I. The first is a generalized sequentail listing of logical operations which are applicable to all graphical configurations. The second is a detailed flow chart for PARAM - 6, so chosen because it includes most of the alternatives presented in this discussion.

A final comment is appropriate concerning the application of computer program PARAMS. As indicated earlier in this article, considerable attention has been given to versatility. The associated price of computational time and memory storage has consequently been paid. The procedures to be used in reducing memory storage for PARAM-1 through PARAM-6 has already been indicated. Compilation time is roughly 1.5 minutes. An additional 30 seconds is consumed for each printout of preliminary graphical information. The actual calculation of 16 curves for each graph requires approximately 3 minutes. Thus a considerable amount of computer time is consumed when a complete set of curves is requested for all possible graphical representations. Various methods exist to reduce this time requirement. By submitted an object deck, compilation time is deleted. This imposes the restriction of submitting a complete data card set for each run, however. Eliminating the input data printout, and the preliminary graphical information printouts will also reduce the time requirements somewhat. This, however, is not recommended because the elaborate information presentation provides an exceedingly valuable check on the validity of the graphical results. Finally, the most time is consumed by a highly sensitive incrementation of the abcissa variable. It will be recalled that the range of the abcissa variable is divided into 900 evenly spaced steps. For each of these abcissa values, a corresponding ordinate value is computed. Although many situations lend themselves to a larger incrementation, it was decided to maintain the fine step sizes for highly sensitive system curves. The maximum sensitivity of 900 points per graph line is the restriction placed upon the program by the CALCOMP plotter in use at the Naval Postgraduate School computer facility. Reduction of this incrementation sensitivity is therefore another means by which computational time can be further reduced. The associated loss of information content must be considered in such cases. As will be pointed out in the next section, not all PARAMS frequency response curves would be required or desired in every design study. The judicious choice of the most valuable plots for a given set of constraints will normally eliminate approximately half of the program possibilities. Only by experience, insight and intelligent design procedures can a good trade-off be achieved between the tools available and the data required.



PARAMETER PLANE FREQUENCY RESPONSE CURVES

FIGURE 3-1



PARAMETER PLANE FREQUENCY RESPONSE CURVES (CONT.)

FIGURE 3-1

4. Tachometer and Acceleration Feedback Compensation Studies.

The mathematical developments of section 2 have provided the basis for constructing the computer programs which were discussed in Section 3. This section will apply these methods to a system study employing tachometer and acceleration feedback as compensation techniques. The frequency response parameter curves will be analyzed according to the respective utility of each as a design tool. Meticulous detail is employed in the initial discussion of these curves. This is to provide a clear understanding of the limitations and advantages of each respective graphical representation.

4.1 Third order

Although an entire library of control engineering studies has been created, there has been no attempt to distinguish a set of example problems as superlative for generalized case studies. Therefore a simple third order system has been considered for an initial application of the parameter plane frequency response curves. The illustrative example which was introduced in section 2 will now be explored in depth. It will be recalled that Figure 2-3 presented the schematic diagram of the third order system with compensation included. The stability curves for the system where mapped into the parameter plane shown in Figure 2-5. In that presentation alpha, the abcissa variable, represented acceleration feedback (K_a). Similarly beta, the ordinate variable, indicated tachometer feedback (K_t). Reference to the characteristic equation of the system shown in Figure 2-3 shows that the steady state velocity error is

$$K_v = \frac{500 + K K_t}{K} = \frac{500 + 100,000\beta}{100,000}$$

Inspection of K_v shows that the smallest allowable value of beta will minimize the steady state velocity error. It is presumed that this is one of the predominant performance criteria. Furthermore, one can arbitrarily stipulate the dominant complex roots to have a damping ratio (ζ) of 0.5. With these system stability requirements, one can see that optimum performance will occur at point A in Figure 2-5. Thus the optimal value of beta is 0.036, while the optimal value of alpha is 0.00047.

A tacit assumption up to this point has been that frequency performance constraints are of minor importance. Most design studies do not conform to this type of ommission; however stability is of interest in all systems. It has therefore been assumed that the two stability performance criteria above are desirable but not mandatory. This permits frequency response specifications to be considered with some initial reference point. Figure 4-1 shows an approximate correlation between frequency response, transient performance, and root locations in the complex frequency plane. Additionally the separate root loci with alpha and beta varied in Figures 4-2 and 4-3 respectively show the approximate variation of root locations as a function of one parameter only. The corresponding effect in the frequency domain is represented by the Bode plots in Figures 4-4 through 4-7. The graphical relationships presented in Figures 4-1 through 4-7 are certainly not new to the design engineer. Rather, they are the fundamental tools used in most system studies today. As such, they are included primarily to establish a firm foundation upon which parameter plane frequency response curves can be built. In this manner, a better understanding can be gained concerning the utility of the latter curves which are unfamiliar to most engineers.

PARAM-1

Complete analysis of the information contained in PARAM-1 plots will now be endeavored. Figure 4-8 shows the relationships between alpha, beta and omega which exist for a constant magnitude of 12 decibels. It is clear from the legend at the bottom of the graph that alpha is the abcissa variable, while beta values are indicated on the ordinate. Each curve in the alpha - beta plane represents a fixed magnitude of 12 decibels, but a different value of omega. Thus the curve of ω = 10 rad/sec barely appears in the lower right hand corner. As omega increases, the curves can be seen to shift toward higher values of beta and lower values of alpha. It can also be seen that the area enclosed by each curve decreases as omega increases to ω = 70 rad/sec. The meaning of each of these 14 curves can be explained as follows: For the third order system in question, any point on a specified omega curve depicts the locus of alpha and beta values which will guarantee a magnitude of 12 decibels at that specific frequency. This by itself does not portray much information, because it says nothing about the area enclosed by the curves, nor the area outside the locus of these curves. In most situations, it is possible to glean considerably more information out of these curves. Consider for a moment that value of alpha and beta at which two omega curves intersect. Point B of Figure 4-8 shows one intersection of the ω = 25 rad/sec and ω = 30 rad/sec curves. For this value of alpha and beta, a magnitude of 12 decibels is guaranteed to exist at both values of omega indicated. This observation is important, for it means that the magnitude is either greater than 12 decibels, or less than that value for the intermediate range of omega. Furthermore, it is possible to assume that the magnitude cannot be both greater than <u>and</u> less than 12 decibels for these intervening values of omega. The reason for this is simply that the magnitude curve is continuous, and must pass through 12 decibels in order to change from a high value to a lower value, or vica versa. If a sufficiently large number of omega curves are drawn in the region of interest, it can be seen that, at Point B, only two omega curves do actually intersect. Therefore, this range of omega values represents the 12 decibel bandwidth of a system which satisfies these values of alpha and beta. With this insight, it can be concluded that a larger bandwidth is obtained for a point slightly below point B. Thus at a value of alpha and beta indicated by point C, interpolation of the omega curves shows an approximate bandwidth from $\omega = 24$ to $\omega = 31$. Conversely, proceding in the opposite direction, the bandwidth is reduced to the point D at which only one frequency curve satisfies the specified value of alpha and beta. No other frequency curves will intersect this point, for it lies on the tangent locus of all frequency curves evaluated at 12 decibels magnitude. Thus this tangent locus defines all the values of alpha and beta which guarantee a peak resonant magnitude of 12 decibels. The value of the resonant peak frequency is not specified by the tangent locus. The reason for this is because omega is an implicit function of the tangent locus equation. Article 2.5 discussed various methods to generate the required non-parametric form of this $M_p(\omega)$ equation, and the limitations encountered in each of those approaches. For those reasons, the tangent locus curve was not constructed. Instead, a sufficiently large number of omega curves were constructed to show the approximate location of the tangent locus. Although the resonant peak frequency is not directly obtainable from the tangent locus, it is clearly seen that such information is readily found by the intersection of a particular omega curve with this tangent line.

The discussion thus far has presumed that all values inside the tangent locus represent peak resonant points greater than 12 decibels. This is intuitively obvious, but no proof has been made of this fact. Figures 4-9 through 4-12 have been included to verify this presumption, and also to illustrate additional phenomena. Figure 4-9 clearly shows that when the magnitude is reduced to 6 decibels, a greater area is enclosed by each of the omega curves. This results in a tangent line representing the locus of peak resonant magnitude equal to 6 decibels. Comparison of this tangent locus with the one in Figure 4-8 will reveal that the 12 decibel tangent locus falls within the area bounded by the omega curves of Figure 4-9. The points B and D of Figure 4-8 have been reproduced on the constant 6 decibel graph for comparison. It can be noted that point B represents a 12 decibel bandwidth of 5 rad/sec and a 6 decibel bandwidth of approximately 13 rad/sec. Additionally, point D depicts a resonant peak magnitude of 12 decibels at 28 rad/sec with a 6 decibel bandwidth of 13 rad/sec. Similar information can be obtained for other values of alpha and beta throughout the parameter plane. By similar analogy, it is possible to determine what range of alpha - beta values will guarantee a peak resonance not to exceed a pre-selected value. Thus all values between the $M_p(\omega) = 12$ and the $M_p(\omega) = 6$ curves will guarantee all magnitudes to be less than 12 decibels, with a 6 decibel bandwidth of variable size depending on the choice of alpha and beta. A closer inspection of Figure 4-9 will reveal that the maximum 6 decibel bandwidth attainable is approximately 17 rad/sec for the situation just described. This maximum bandwidth occurs approximately alpha equal to 0.0001 and beta equal to 0.018. At this point the resonant peak magnitude is slightly less than 12 decibels, with a resonant angular frequency of approximately 40 rad/sec. The cut-off frequencies occur at ω = 32 rad/sec and ω = 49 rad/sec. All of the above information has been derived from the evaluation of two PARAM-1 graphical plots. It will be recalled that the PARAM-7 curves of Figures 4-4 through 4-7 merely portrayed Bode curves as a function of one parameter only. These PARAM-1 curves denote a portion of many Bode curves as a function of two system parameters. The value of these two is restricted to only two magnitudes, but certainly the analysis of any two such graphs provides a certain insight as to the system behavior for other magnitudes. Interpolation can be used for the intermediate magnitude range. Such a procedure is not recommended without some caution for extensions of magnitude studies outside the graphed limits. The consideration of Figures 4-10 through 4-12 will illustrate this point more clearly. For a constant magnitude of 3 decibels, Figure 4-10 shows a dramatic increase in the applicable range of alpha - beta values. The graph scales of all PARAM-1 plots were kept the same simply to illustrate the large increase in area encompassed by each constant frequency curve as the value of magnitude is decreased. As might be anticipated, a much larger 3 decibel bandwidth exists for the points, B, C and D shown on Figure 4-8. The bandwidth of these three points do not vary greatly, however. Each is approximately 20 rad/sec. This illustrates the sensitivity of beta, the tachometer feedback in the area under discussion. For a given value of alpha equal to 0.008, a slight change in beta greatly affects the <u>size</u> of the peak reasonant magnitude. Conversely, an equal change in alpha reflects a vast difference in omega, the <u>location</u> of this peak resonant magnitude, plus a slight change in its magnitude value. Although the sensitivity of the alpha variable might not seem apparent by looking at the curves, it must be remembered that there is a different graph scale for each of the axis variables.

Figures 4-11 and 4-12 show most clearly why the analysis of merely two PARAM-1 frequency response representations might be misleading. Consider point B of Figure 4-8 which is reproduced on the zero decibel magnitude graph of Figure 4-11. The constant omega curves of this plot show that no two curves intersect at point B. Clearly, it is seen that all values of omega less than 40 rad/sec encompass this choice of alpha and beta. Therefore a cut-off frequency of approximately 39 rad/sec is obtained for the zero decibel bandwidth of that system. Similarly, from Figure 4-12, the -3 decibel bandwidth is about 43 rad/sec. Using only these latter two PARAM-1 plots, one cannot discern any resonant peak for the system frequency response associated with these values of alpha and beta. It is only through the apriori information portrayed in any one of the Figures 4-8 through 4-10 by which we are able to make such a statement that a resonant magnitude actually exists. Only by the combined interpolation of all the Figures 4-8 through 4-10, can we state with a fair degree of confidence that the value of this resonant magnitude is approximately 15 decibels. This of course could be checked by constructing another PARAM-1 curve, or perhaps a Bode plot. Neither of these are necessary for initial coarse approximations. It will soon be shown that the combined usage of PARAM-1 and PARAM-2 provides a much better method. The reason for the inclusion of so many PARAM-1 curves was simply to illustrate how a valuable insight can be obtained for system frequency response studies when considering two parameter variations simultaneously. Within the limitations just indicated, it is possible to determine how two parameter variations affect bandwidth and peak resonance from a careful inspection of several PARAM-1 graphs.

Before proceding to PARAM-2 graphical configurations, it will be advantageous to review Figures 4-8 through 4-12 in a three dimensional concept. Consider for a moment a curved conical surface similar in shape to a bull's horn. Also, let this curved cone be elliptical in shape, rather than round. Next assume that the base of this surface is in the lower right hand corner, nearest to the observer. The smaller end of this surface would therefore be in the upper left hand corner of the observation sector, at a certain distance away from the observer. Now superimpose the omega axis on the centerline of this conical surface in such a manner that the low values of omega are at the base of this surface, or nearest to the observation point. The higher values of omega would consequently lie further away from this observation point, increasing in value toward the curved conical tip. With this three dimensional surface concept in mind, look at Figure 4-8. The curved omega axis originates in the lower right hand corner and extends through the alpha - beta plane, curving upward and slightly to the right. Figure 4-9 depicts this curved conical surface as larger in size, (i.e., closer to the observer), with the base of this surface shifted slightly to the left, thereby bringing it more into the line of sight. Bringing this curved conical surface still closer to the observation point makes it appear larger, as seen in Figure 4-10. The zero decibel PARAM-1 graph, Figure 4-11, portrays an even larger shape with the base shifted even more to the left. Finally Figure 4-12 can be regarded observing this curved surface with its base very close, similar in appearance to passing through a curved tunnel under a river.

The preceding illustrative example can be considered as elementary, but is fundamental to the understanding of the various graphical representations to follow. No attempt will be made to correlate the illustration to the PARAM-7 curves of Figures 4-4 through 4-7, since those are essentially single parameter variations of the series of two parameter surfaces just described. Better use of the illustrative example is made by correlating PARAM-1 with PARAM-2.

PARAM-2.

Consider now a cross sectional cut of the curved conical surface described above. PARAM-2 effectively maps constant magnitude curves onto the alpha - beta parameter plane for a fixed value of angular frequency. In this manner, it is possible to show how a particular omega value, such as ω = 10 rad/sec will vary as a function of magnitude in the alpha - beta plane. Consider again the ω = 10 rad/sec curve in Figures 4-8 through 4-11. The associated PARAM-2 plot is shown in Figure 4-13. Since the ω = 10 rad/sec curve represented a cross section of the curved conical surface near its base, it is possible to observe how this base is rotated toward the left as magnitude is decreased. Further inspection of Figure 4-13 shows how the curve increases in size with a reduction of magnitude. The -3 decibel, ω = 10 rad/sec curve is in fact shifted completely off the graph of Figure 4-13. This effectively shows that no alpha - beta value within the prescribed range will have a magnitude of -3 decibels at an angular frequency of 10 rad/sec. No information is conveyed about any other angular frequency in Figure 4-13. To obtain such information would necessitate a different PARAM-2 plot, or a comparison with one or more PARAM-1 curves. This does not preclude the correlation of other frequency response curves. They can be consulted for pertinent relationships, and will be discussed later. Figure 4-14 is a graphical representation of constant magnitude curves for a fixed angular frequency of 20 rad/sec. Inspection of these curves shows that they lie very closely to the centroid of the curved conical surface discussed in the illustrative example of this article. This can be verified by their concentric shape. Due to the graphical construction technique, these curves do not extend to the bottom of the graph. This can be explained by the fact that these curves are almost vertical at the lower graph edge. Even though the abcissa variable is incremented 900 times during each curve computation, this is not sensitive enough to allow the detection of such a steep slope. If an infinitessimally small step size were possible, these curve values would be detected. This limitation of the graphical plotting procedure must be kept in mind. The mathematical theory of section 2 indicated that two quadratic formula solutions were necessary for the complete solution of any curve generated by PARAM-1 through PARAM-6. The computer technique discussion of section 3 discussed how each graph curve was constructed first for one sign of the square root coefficient in the quadratic formula, and then for the other. Subsequent graphs will show how these two curve solutions do not always connect. The reason for this again lies in the finite incrementation of the abcissa variable. Returning once again to Figure 4-14, it is possible to note certain characteristics regarding the angular frequency of 20 rad/sec as a function of the two parameters alpha and beta. The intersection of the peak values of each magnitude curve defines a vertical line in the parameter plane. Therefore it is possible to conclude that variations in magnitude for an angular frequency of 20 rad/sec are highly sensitive to beta values, and quite insensitive to changes in alpha. This says nothing about resonance magnitudes. Rather, each curve guarantees a particular magnitude value for a given angular frequency. Also, each curve defines an alpha - beta region within which the magnitude will always be greater than the boundary value for that value of omega. As a further analysis of the system response for point B of Figure 4-8, it can be seen that such a system will have a magnitude of 6 decibels at a frequency of 20 rad/sec. Figures 4-15 and 4-16 show considerably more information than the previous two PARAM-2 graphs, because the center of these eliptical curves is indicated. This center corresponds to the centerline of the three dimensional curved conical surface which has been repeatedly referenced up to this point. It should be observed that the center of these ellipses in Figures 4-15 and 4-16 denote the maximum magnitudes attainable at their respective angular frequencies. Thus by analogy, the centerline of the three dimensional surfaces in Figures 4-8 through 4-16 serves a dual purpose. Not only does it specify a particular angular frequency, but it denotes the locus of the highest peak resonant magnitudes for any alpha - beta combination of the system! This is an important conclusion, because it indicates whether a desired peak resonant magnitude value can <u>ever</u> be reached for the system under consideration. The comparison of PARAM-1 and PARAM-2 graphs further shows how the conical surfaces expand about this centerline with decreasing magnitude. From these, it is possible to visualize the locus of constant magnitude conical surfaces, each surrounding the centerline just discussed. It is therefore seen that a reduction of tachometer gain coupled with an increase in acceleration gain will shift the resonance peak to a higher frequency. Point B of Figure 4-8 has also been reproduced on Figures 4-15 and 4-16. These show that a system with the indicated alpha - beta values will have a magnitude of 12 decibels at $\omega = 30$ rad/sec, and -1 decibels at an angular frequency of 40 rad/sec. At this point, enough information has been obtained to obtain a fairly accurate Bode plot for the parameter values given at point B. Further comparison of the 15 decibel magnitude curve in Figures 4-14 through 4-16 show that point B does in fact have a magnitude equal to 15 decibels. This confirms the assumption of this value based on interpolation of the PARAM-1 curves alone. Since enough information has been depicted in the PARAM-1 and PARAM-2 curves to portray a Bode plot of point B, it is obvious that similar Bode plots could be obtained for any other desired combination of alpha-beta points. This is exceedingly valuable, because it allows the design engineer to construct any number of coarse Bode plots by evaluating the data contained in Figures 4-8 through 4-16. Therefore it is concluded that the combined utilization of PARAM-1 and PARAM-2 is a fundamental starting point for all two parameter frequency response design studies.

The preceding discussion concerning the use of PARAM-1 and PARAM-2 was very comprehensive because of the following three key factors: First, a thorough understanding of new concepts can only be gained by a slow and detailed presentation of the uses and limitations of such innovations. Secondly, the combined application of PARAM-1 and PARAM-2 plots provides the broadest information spectrum for frequency response design studies. Finally, a thorough understanding of these two parameter plots is prerequisite to visualizing the following frequency response projections which are essentially a function of one system parameter. The remainder of this article will cover the various roles played by PARAM-3 through PARAM-6.

PARAM-3.

Section 3 developed the major graphical presentations which were considered to be of most use to design engineer. In that discussion, it was indicated that two basically different configurations were available in PARAM-3. Figure 3-1 showed in schematic form these differences. Attention will first be focused on the MODORD=1 variety as shown in Figure 4-17. Before discussing the information content of this graph, an apparent mistake in the graphical plotting procedure is noteworthy. This set of curves was purposely offset to illustrate a limitation encountered by careless selection of axis variable ranges. Graph scales are limited to one significant figure. As such the automatic scaling of the program PARAMS does not always produce the values which might be expected. The requested range of alpha values was from -0.001 to 0.007. Since the ordinate length was specified as ten inches, the program computed a grid scale of 0.0008 units per inch for alpha. Accordingly this scale was used; however, it placed the origin one inch above the minimum value in accordance with program plotting limitations. Thus the requested range of alpha values was plotted properly, even though the choice of ordinate values was improperly chosen. The preceding was for information purposes only; PARAM-3 plots will now be studied.

Figure 4-17 portrays a set of constant beta curves on the magnitude - alpha plane for a given fixed angular frequency value of ω = 30 rad/sec. This graph can best be understood by comparing it with the PARAM-2 plot of the same constant omega, namely Figure 4-15. Effectively, the PARAM-3, MODORD=1 curves take the information contained in the horizontal cross-sections of PARAM-2 and map this data onto the magnitude - alpha plane. This can be most easily visualized by taking the base of Figure 4-15 and comparing it with the beta = 0.0 curve in Figure 4-17. With these two viewpoints, one readily sees another facet of PARAM-2 plots. Consider for a moment a topographical map of land terrain. Such maps represent constant altitude as a continuous curve. Each such curve is always a closed contour, with either peaks or valleys contained within the curve boundaries. PARAM-2 is such a topographical plot, with magnitude equal to elevation. This concept is borne out by the magnitude-alpha plane of Figure 4-17. The constant beta curves of this latter graph can be regarded as vertical slices into the topographical hill-side. As such, one sees which cross sections have actually been sliced out of the PARAM-2 magnitude contours. The complete range of PARAM-2 alpha and beta values has been spanned by the ordinate range and constant beta curves in Figure 4-17. Accordingly one might conclude that a good indication of the system's magnitude behavior has been conveyed by the PARAM-3, MODORD=1 graph for ω = 30 rad/sec. This is most emphatically not true! Analysis of Figure 4-17 indicates that the greatest magnitude occurs at beta = 0.0 and that the magnitude contours decrease as beta approaches 0.05. Careful review of Figure 4-15 will disclose a serious fallacy in such a conclusion. Figure 4-17 shows no indication of the magnitude peak resonance at beta = 0.004. Without this apriori knowledge, a completely erroneous interpretation would have been made by analyzing the curves in PARAM-3, MODORD=1. This example stresses once again the importance of viewing system frequency response as a function of the two system parameters.

Consideration will now be given to the alternate PARAM-3 curve designated by MODORD=2. This graph presents constant alpha curves in the beta magnitude plane for a fixed omega. As such, these curves simply slice vertical cross-sectional areas out of the magnitude contours of PARAM-2. Figure 4-18 shows such a set of curves for a constant angular frequency of 30 rad/sec. Looking at Figure 4-15 again, all curve values lie within a fairly small range of alpha values. Accordingly, one would only expect graph curves for alpha less than 0.0021 to lie in the beta - magnitude plane for magnitudes greater than or equal to -3 decibels. Furthermore one would expect a series of magnitude peaks to occur at beta = .004 with the maximum amgnitude occuring on the alpha = 0.0005 curve. These factors are verified in Figure 4-18 where the alpha = 0.0005 curve has maximum magnitude greatly in excess of 21 decibels. All other curves represent lower magnitude ranges as was anticipated. In this set of alpha curves, the maximum magnitude curve was intentionally constructed to indicate the large magnitude at peak resonance. This curve might have been overlooked, were it not for the apriori knowledge gained from Figure 4-15.

The preceding discussion of PARAM-3 curves showed serious limitations in their usage when the PARAM-2 curve of the same constant omega is not utilized. When used in conjunction with PARAM-2 graphs, however, the PARAM-3 curves do provide some useful information. They are particularly useful for the area enclosed by the largest value of magnitude in the PARAM-2 plots. This area tells nothing about the peak resonant magnitude, other than the frequency at which it occurs. By constructing one PARAM-3 curve along the bisector of these magnitude curves, one quickly gains the value of peak magnitude and the magnitude sensitivity about this resonant point. Should the precise peak magnitude location not be so obvious on other PARAM-2 plots, a series of curves in the PARAM-3 graph will clearly indicate its location. Due to the elliptical shape of the PARAM-2 magnitude curves for this system, the PARAM-3, MODORD=2 configuration would be better suited than the PARAM-3, MODORD=1 graph. The reason for this is simply that the magnitude is more sensitive to alpha than beta. Therefore a much smaller range of alpha values need be considered than would be required for beta curves.

Up to this point, only one value of omega has been discussed. The PARAM-3 curves for other angular frequencies can be visualized by studying Figures 4-13, 4-14, and 4-16. The PARAM-2 plot of Figure 4-13 shows a very gentle magnitude slope along constant alpha and constant beta lines. Thus for constant alpha lines, one would expect the magnitude to decrease slowly with an increase in beta. The magnitude would decrease more rapidly for larger constant alpha curve values than they would for smaller curve values. Conversely, for constant beta lines, it is seen that magnitude will increase slowly as alpha is increased. Large constant beta curve values would show a slower increase in magnitude than small beta curve values. Figures 4-19 and 4-20 are included for verification of these trends.

Inspection of Figure 4-14 shows that the PARAM-3, MODOR=1 graph will have a maximum magnitude at alpha = 0.0018 for all beta curves. The maximum value of magnitude for any value of alpha will be obtained on the beta = 0 curve. All other beta curves will lie inside this magnitude boundary. The shpae of these curves will be similar to those shown on Figure 4-18. Since the magnitude is less sensitive at 20 rad/sec than at 30 rad/sec, the curves will be less pointed. The PARAM-3, MODORD-2 curves will not be of the shape just discussed. This is because the vertical cross-sections of the PARAM-2 plot show a decrease in magnitude as beta increases. All alpha curves will lie inside the constant alpha curve equal to 0.0018, for this curve represents a ridge of maximum magnitude. The constant ω = 40 rad/sec magnitude curves of Figure 4-16 show that the peak resonance occurs on the beta axis at an alpha value of approximately 0.012. The sensitivity of magnitude is greatest in this graph. Accordingly, the PARAM-3, MODORD=1 curves will be much more pointed than those shown in Figure 4-17. Also the center of these magnitude peaks will be shifted down to the abcissa. The PARAM-3, MODORD=2 curves will be likewise sharper than those shown in Figure 4-18, but this time the centerline of the magnitude peaks will lie on the beta = 0.012 line.

It has been shown that for this system the PARAM-3 curves are of limited use. Detailed correlation between PARAM-2 and PARAM-3 plots was made to show the inter-relationships which exist. The reason for discussing PARAM-3 graphs was to show how such information can be picked off of the PARAM-2 plots, thereby eliminating the requirement for another graphical presentation. A more important reason for this detailed analysis was to point out the correlations applicable to this system under study. Many systems will undoubtedly be more complex than this. Such cases might require the use of these curves. Therefore it is essential to obtain a firm understanding of these graphical advantages and limitations.

PARAM-4.

The graphical concepts of PARAM-4 will now be discussed for the third order system under consideration. Again, two basically different plots are obtained for this program. The PARAM-4, MODORD=1 configuration graphs constant magnitude curves onto the alpha - omega plane for a fixed value of beta. Figure 4-21 shows one such plot for a given fixed beta value of 0.036. At first glance, these curves seem very strange. Once again, however, recourse to the two parameter frequency response curves of PARAM-1 will show a striking relationship. Recall for a moment the three dimensional concept that was introduced in the study of PARAM-1 graphs. Each three dimensional curved elliptical cone shaped surface was a function of alpha, beta, and omega for a given value of magnitude. As magnitude was decreased, the conical surface base was observed to rotate into the observation line of sight, while the overall size expanded. The simultaneous rotation and expansion caused the base portion to expand more than the smaller end. Furthermore, recall that omega increased in value from the base toward the small end. The PARAM-4, MODORD=1 graph effectively slices a horizontal cross section out of this PARAM-1 three dimensional surface at the prescribed value of beta. To see this more clearly, Figures 4-8 through 4-12 will be reviewed in order. Consider first the constant decibel magnitude PARAM-1 graph of Figure 4-8. By cutting a perpendicular cross section along the beta = 0.036 line, it is seen that only one omega curve is severed namely the ω = 65 rad/sec curve. Note, however, that omega is a continuous elliptical surface; only a small number of cross sections are depicted in Figure 4-8. By interpolation, it can be seen that a perpendicular cross section of PARAM-1 will intersect the range of omega values between approximately 62 through 67. Furthermore note that no other omega curves have been severed in this manner. This intersection occurs at a range of alpha values between -0.002 and -0.004. Now turn to Figure 4-21 and inspect the 12 decibel magnitude curves reproduced there. By a similar analysis of Figures 4-9 through 4-12, it is possible to establish a positive correlation between the PARAM-1 and PARAM-4, MODORD=1 graphs. Next consider the PARAM-4, MODORD=2 plot shown in Figure 4-22. This configuration graphs constant magnitude curves onto the beta - omega plane for a fixed value of alpha. By methods analogous to those presented above, it is possible to slice a vertical cross section of the PARAM-1 graph at the prescribed value of alpha. The information thus obtained is seen to appear as PARAM-4, MODORD=2. The PARAM-4 curves are effectively topographical maps with magnitude equivalent to elevation. Their utility lies in the ability to correlate magnitude, frequency and one system parameter. They are restricted by the second system parameter which has been held at a constant value. As such they would only be suited for supplying additional frequency response information after an initial choice of one variable has been made. Providing a sufficient number of magnitude curves have been constructed on these graphs, they become in essence a locus of Bode plots. When used in this manner, they become valuable tools. The constant alpha and beta values chosen for figures 4-21 and 4-22 were so chosen because they coincided with the values used in figures 4-4 through 4-7. Thus a comparison of these PARAM-4 curves with the familiar Bode magnitude plots can easily be made. The PARAM-4 curves can therefore be regarded as most useful for coarse Bode plot studies, since they portray magnitude-frequency information for a much larger number of alpha or beta values than would be feasible in a conventional Bode plot study. In this manner a design engineer would most likely construct enough PARAM-1 and PARAM-2 plots to make a judicuous first approximation for each of the system parameters. Using these, he could then construct both PARAM-4 plots and check his system performance with a fair degree of accuracy. After suitable corrections, he would then use PARAM-7 for a final refined system frequency response performance check.

PARAM-5

The discussions of PARAM-3 and PARAM-4 have considered cross sectional cuts of the PARAM-2 and PARAM-1 planes respectively. A slightly different concept is portrayed in PARAM-5 and PARAM-6. Consider first the PARAM-5, MODORD=1 graph shown in figure 4-23. This shows a series of omega curves plotted upon the alpha magnitude plane for a fixed value of beta = 0.025. Each constant magnitude depicts a horizontal cross section of the PARAM-1 graph corresponding to that constant magnitude. The location of the horiztonal cross-section is fixed by the constant beta = 0.025 for the PARAM-5 plot. This is most clearly shown by the following comparison. Arbitrarily choose the constant magnitude line of 6 decibels. This automatically selectes the PARAM-1 plot shown in figure 4-9. Since beta is fixed at a value of 0.025, this is the horizontal line of figure 4-9 to be considered. Positive correlation between the omega curves crossing this beta = 0.025 line in figure 4.9 is thereby made with those omega curves crossing the magnitude line of 6 decibels in figure 4-23. Effectively the PARAM-5, MODORD graph shows the series of PARAM-1 plots for any given constant beta value. Merely studying the different magnitude lines, one can see how a constant beta cross section would look on different PARAM-1 plots. The PARAM-5, MODORD=2 graphs show this same behavior, with the roles of alpha and beta interchanged.

A comparison can also be made with a series of PARAM-2 plots, since they represent another three dimensional surface projected into the alpha beta parameter plane. In that case, however, each omega curve in the PARAM-5 graph represents a different PARAM-2 plot. Consider for a moment the ω=20 rad/sec curve of figure 4-23. This curve depicts the topological relief of a cross section of the PARAM-2 magnitude contour graph when that cross section occurs at beta = 0.025. Similarly, different omega curves of figure 4-23 represent such a cross section in other PARAM-2 plots.

The correlation between PARAM-5 and PARAM-1 as well as PARAM-2 provides a rather important conclusion. One normally wants to get a good indication of system behavior by simultaneously varying alpha and beta. It is not necessary to construct a large number of PARAM-1 plots to do this. Neither is it necessary to construct a large number of PARAM-2 plots. By constructing only a few PARAM-1 graphs, as well as two or at most three PARAM-2 graphs, one can gain valuable insight into the system behavior in the voids by studying the PARAM-5 plots. Thus for an initial system design study, one might be inclined to use the following six graphs: two PARAM-1 plots, two PARAM-2 plots, and two PARAM-5 plots. The constant magnitude values of the PARAM-1 plots would most likely be chosen from the bandwidth and resonant peak criteria. The constant omega values for the two PARAM-2 plots would normally be dictated by the cut-off frequencies. Alternatly, the peak resonant frequency might be used as a governing choice. By choosing an intermediate value of alpha and beta, the PARAM-5 curves would indicate how other PARAM-1 and PARAM-2 graphs would have appeared. For illustrative purposes, suppose a problem called for a -3 decibel bandwidth of 60 rad/sec, with a peak resonance magnitude of no more than 6 decibels around 40 rad/sec. Construction of figures 4-9, 4-12, and 4-16, one would be able to conclude the following information:

1) The tangent locus of the omega curves in figure 4-9 define the limiting values of alpha and beta as dictated by the resonance magnitude constraint.
2) The intersection of the tangent locus and the constant ω = 40 rad/sec curve will determine which values of alpha and beta will give this desired resonant frequency.
3) Looking at figure 4-12, one sees that the alpha and beta values determined above will not provide a -3 decibel bandwidth of 60 rad/sec. Instead, it can be seen by interpolation that the bandwidth will be approximately 57 rad/sec.
4) Figure 4-16 shows the magnitude relationship at the desired resonant frequency. Thus by studying this graph, one sees that a resonant magnitude of approximately 7 decibels will result from a 60 rad/sec bandwidth.

The design example above has shown a common problem which exists for most engineering studies. The desired performance measures cannot be attained; instead some compromise must be made. This conclusion was derived by simply observing three frequency response parameter plane graphs. The advantage of PARAM-5 graphs lies in the ability to correlate performance sensitivities with parameter variations. Even though the curves shown in figure 4-23 do not coincide with the beta most likely to be selected, they do illustrate the behavior of magnitude and omega at a neighboring beta value. Thus it is possible to project that information onto one of the PARAM-1, PARAM-2 graphs and extrapolate the data thus obtained. Additionally, one can do the same with the PARAM-5 curves shown in figure 4-24. This figure shows the magnitude and omega variations as a function of beta for a constant alpha also near the point of interest. In effect the curves in figure 4-24 reflect the behavior about the peak resonant point of ω=35 rad/sec. It is seen that a variation in beta will increase the resonant magnitude for smaller beta values, with a corresponding decrease in magnitude for higher beta values. Figure 4-24 shows another important factor. At beta = 0.036 one sees that a flat frequency response of 0 decibels will be obtained for a bandwidth of approximately 35 rad/sec. This can be verified in figures 4-11 and 4-25.

Figures 4-25 and 4-26 have been included to show more vividly how the interpolation of PARAM-5 plots might be made. Consider the PARAM-5 graphs as topological portrayals of omega. Figure 4-25 shows how these omega contours change from those of Figure 4-23 as beta increases. Note that all omega curves are reduced in magnitude when beta is increased. Note further that a reduction in the number of omega curves will result in a loss of important information. The eight curves shown in figure 4-25 might lead the observer to conclude that magnitude decreases continually as alpha is decreased. Only by careful inspection of the ω = 40 rad/sec curve can one see that this is not the case. By inspecting the complete array of curves in figure 4-23, however, one sees a resonance trend for negative alpha values. In this figure, resonance occurs at approximately 55 rad/sec. Using this information, another PARAM-2 graph can be visualized for constant ω = 55 rad/sec. The curves would be similar in shape to those shown in figure 4-15 and 4-16, but smaller in size. The center of these elliptical curves would occur at beta = 0.036 and alpha = -0.003. The analogous procedure could also be employed using either figures 4-24 or 4-26.

PARAM-6

The discussions of PARAM-3 through PARAM-5 have shown how various graphical configurations result from taking cross sectional cuts in the PARAM-1 and PARAM-2 frequency response parameter plane curves. The curves shown in PARAM-6 depict another such cross sectional projection. Consider first the curves shown in figure 4-27. They correspond to constant beta curves mapped onto the alpha-omega plane for a given preselected magnitude value of 12 decibels. Accordingly, this graph corresponds to the PARAM-1 plot shown in figure 4-8. The full utilization of the entire set of graphs will now become clear. Effectively the PARAM-6 graphs show the third variable of the curved elliptical surface which was projected onto the parameter plane in PARAM-1. In this manner, the third variable, omega, now becomes one of the axis variables while cross sectional cuts of the three dimensional alpha-beta-omega surface are obtained by slicing down along constant beta lines or constant alpha lines. The latter cross sections are pictured in figure 4-28. Thus when the corresponding PARAM-1 and PARAM-6 graphs are viewed simultaneously, one can visualize the three dimensional alpha-beta-omega surface in its true form. The illustrative example of an elliptically shape bull's horn is applicable in all but one facet which will now be corrected. It was originally stipulated that the omega axis passed down the conical surface, and was rotated and shifted slightly as magnitude decreased. This stipulation was made to simplify the visualization of the surface. Inspection of the PARAM-6 curves will show a slight modification of this illustration. The omega axis does <u>not</u> pass down the surface, but instead projects perpendicularly through the origin of the PARAM-1 plots. In this manner it is seen that the surface has shifted slightly, and been rotated in the three dimensional alpha-beta-omega space! Now the illustration is correct in its entirety. Analyzing the corresponding PARAM-1 and PARAM-6 graphs gives a complete description of the portrayed surface in three dimensions. Inspection of any one of the PARAM-2 plots reveals the change in shape of this three dimensional surface as the fourth dimensional parameter, magnitude, is varied. Figures 4-29 through 4-32 are included for amplification and verification of the above results.

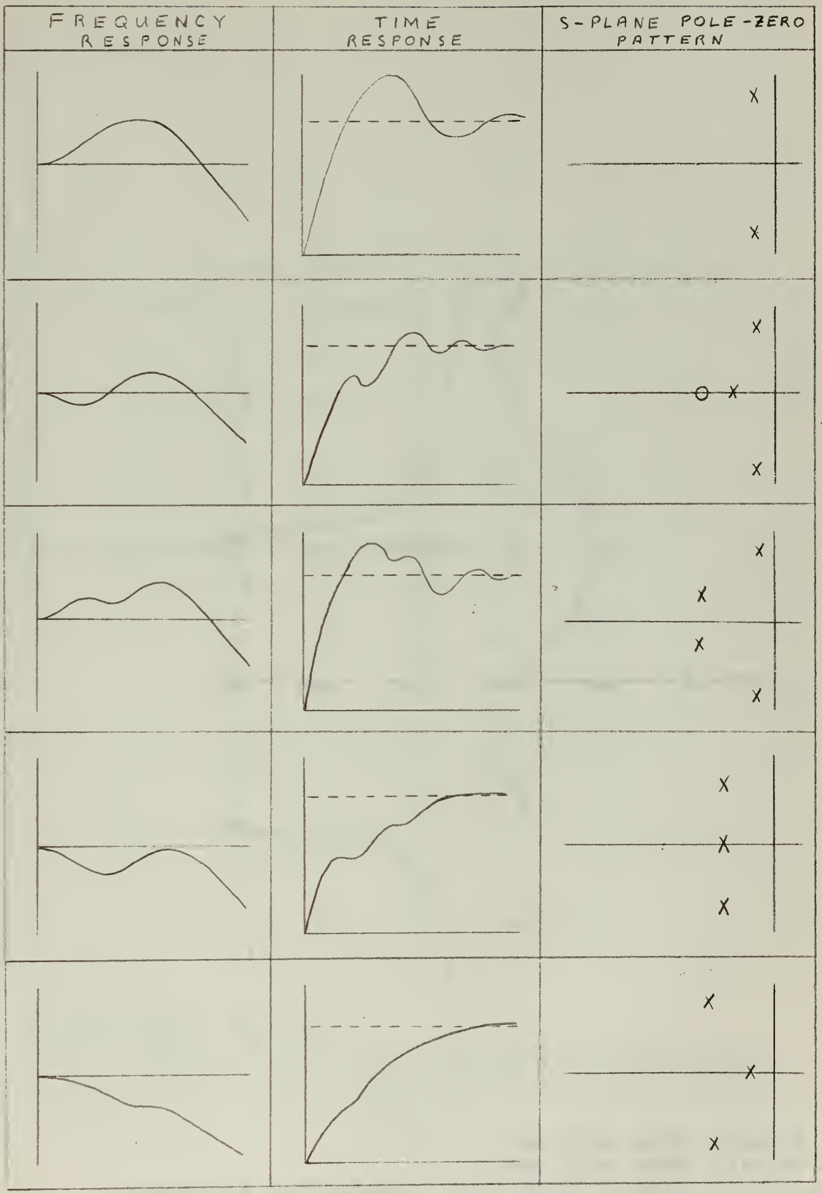
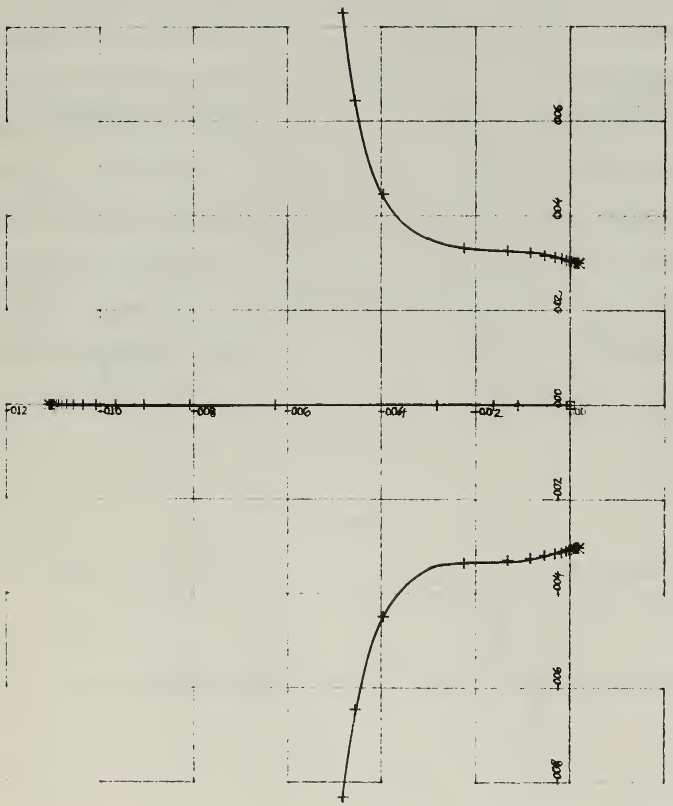
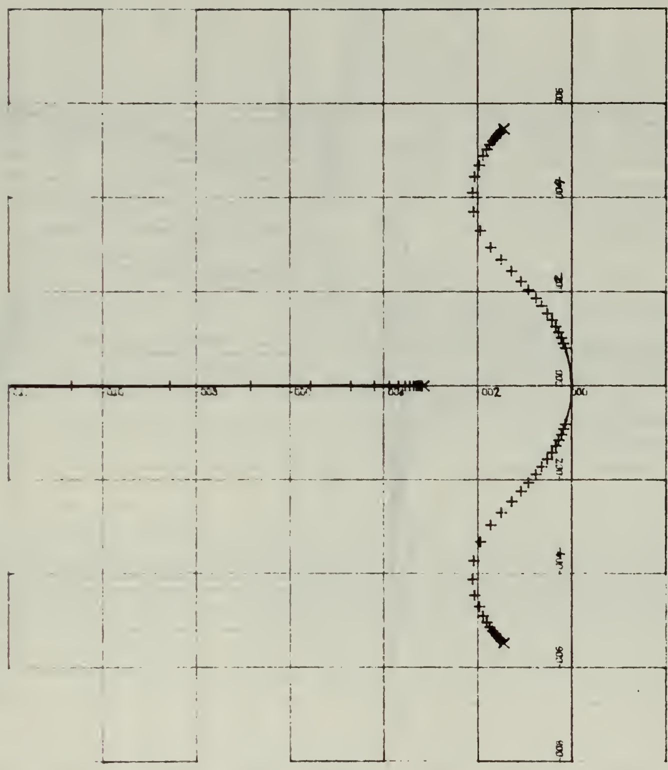

FREQUENCY RESPONSE
TIME RESPONSE
S-PLANE POLE-ZERO PATTERN

FIGURE 4-1



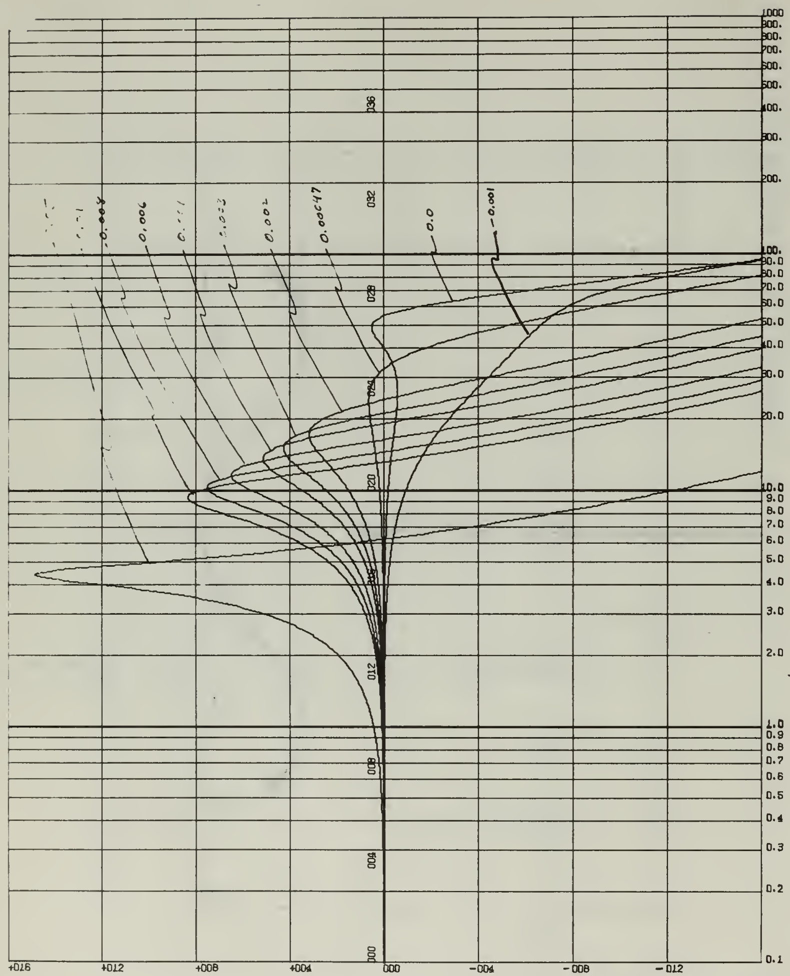
X-SCALE=2.00E+01 UNITS INCH.
Y-SCALE=2.00E+01 UNITS INCH.
GLAVIS,ROOT LOCUS FOR TACHOMETER FEEDBACK
S**3+100S**2+(500+100,000K)S+100,000=0; 0≤K≤0.02

FIGURE 4-2



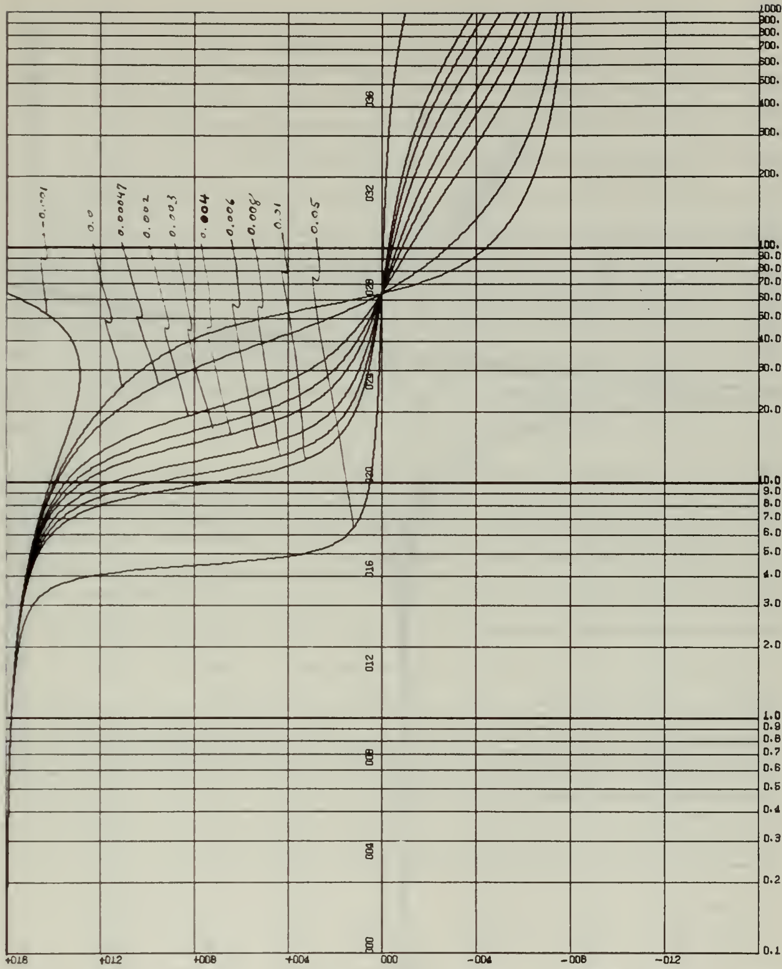
X-SCALE=2.00E+01 UNITS INCH.
Y-SCALE=2.00E+01 UNITS INCH.
LAHIS, ROOT LOCUS FOR ACCELERATION FEEDBACK
S**3+(60+100,000K)S**2+4100S+100,000=0, 0≤K≤0..1

FIGURE 4-3



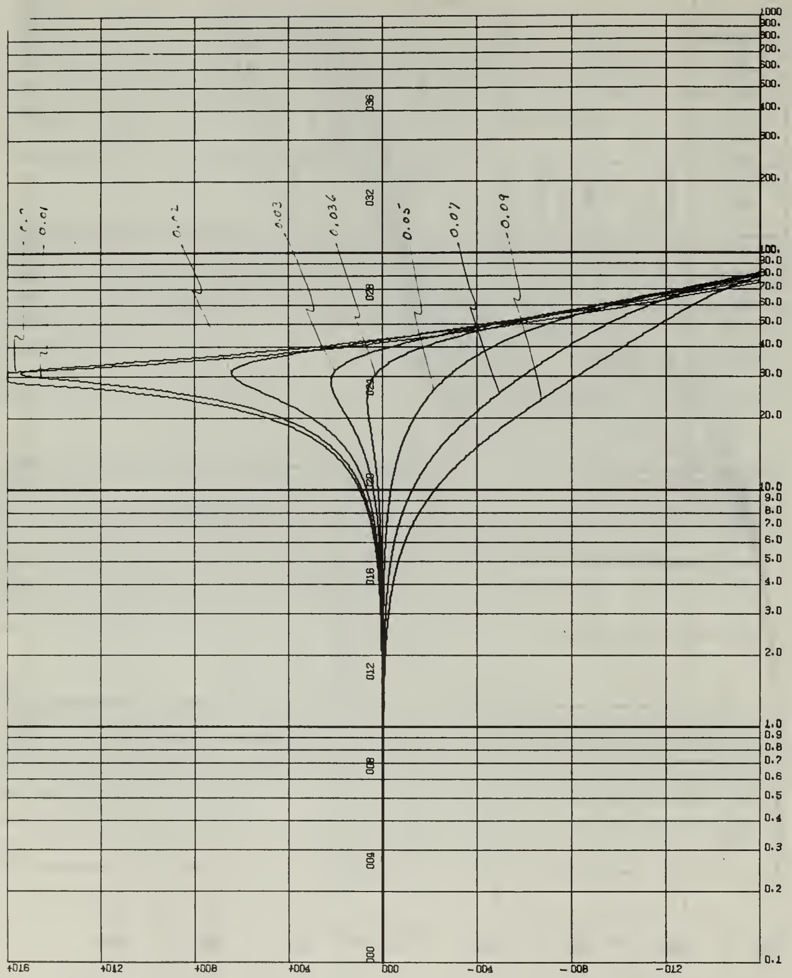
X-SCALE=4.00E+00 UNITS INCH.
Y-SCALE=4.00E-01 UNITS INCH.
GLAVIS, PARAM 7, 3RD ØRDER T/A F.B.,ALPHA CURVES
ØRDINATE=MAGNITUDE, ABCISSA=ØMEGA, CØNBETA=0.036

FIGURE 4-4



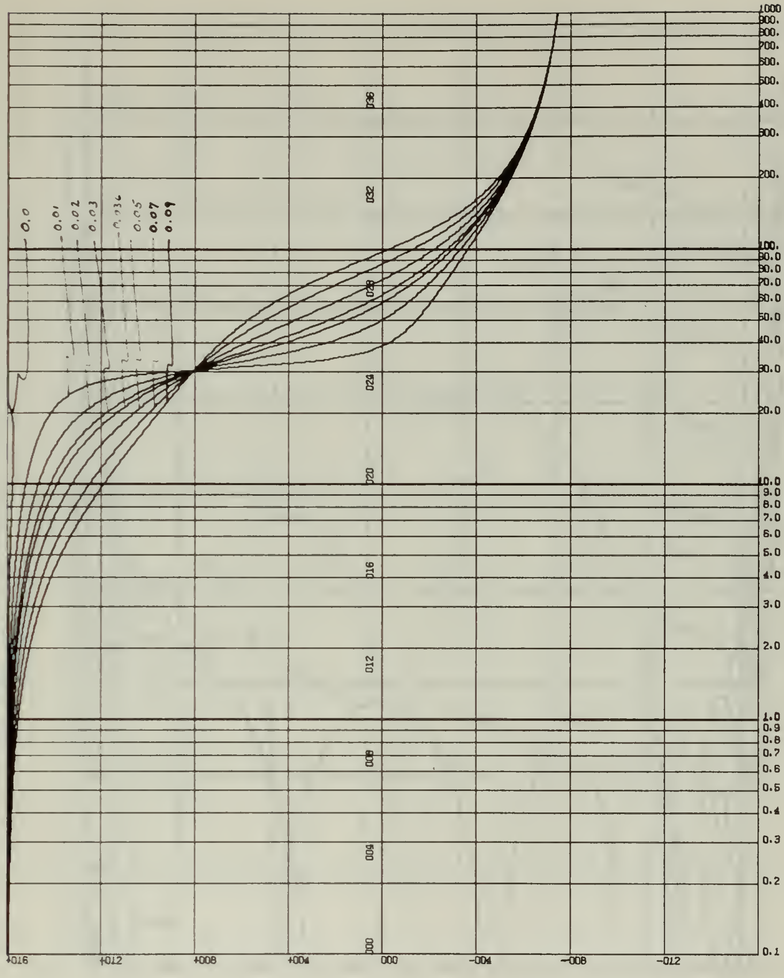
' -SCALE=4.00E+00 UNITS INCH.
Y-SCALE=4.00E-01 UNITS INCH.
GLAVIS, PARAM 7, 3RD ØRDER T/A F.B.,ALPHA CURVES
ØRDINATE=PHASE ,   ABCISSA=ØMEGA, CØNBETA=0.036

FIGURE 4-5



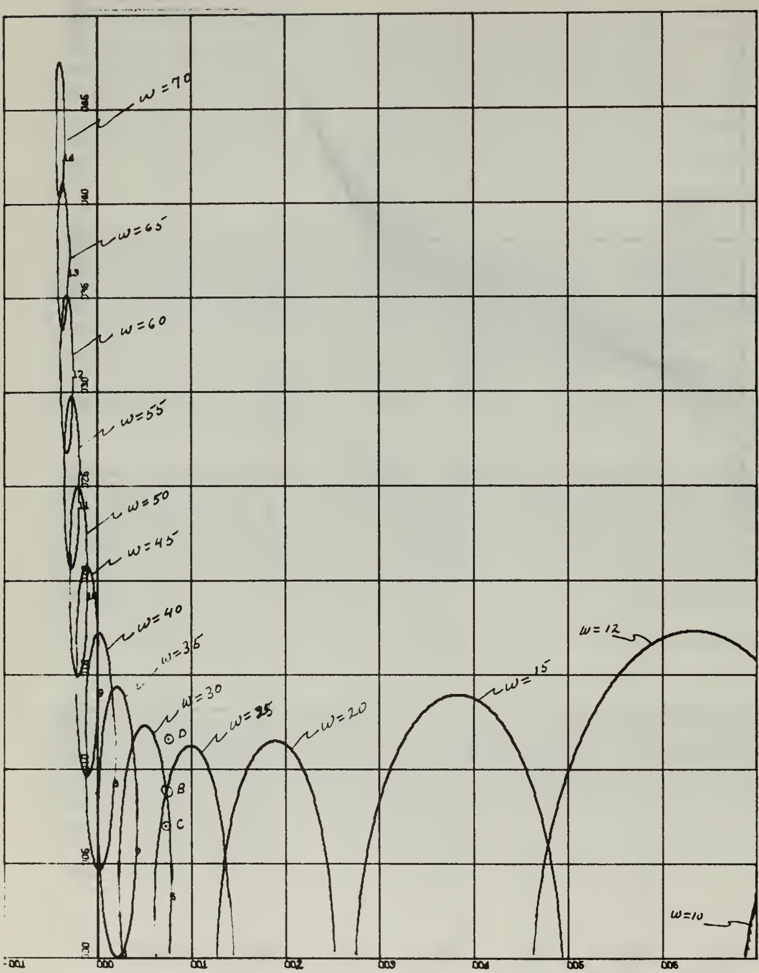
X-SCALE=4.00E+00 UNITS INCH.
Y-SCALE=4.00E-01 UNITS INCH.
GLAVIS, PARAM 7, 3RD ØRDER T/A F.B., BETA CURVES
ØRDINATE=MAGNITUDE,ABCISSA=ØMEGA,CØNALPHA=.00047

FIGURE 4-6



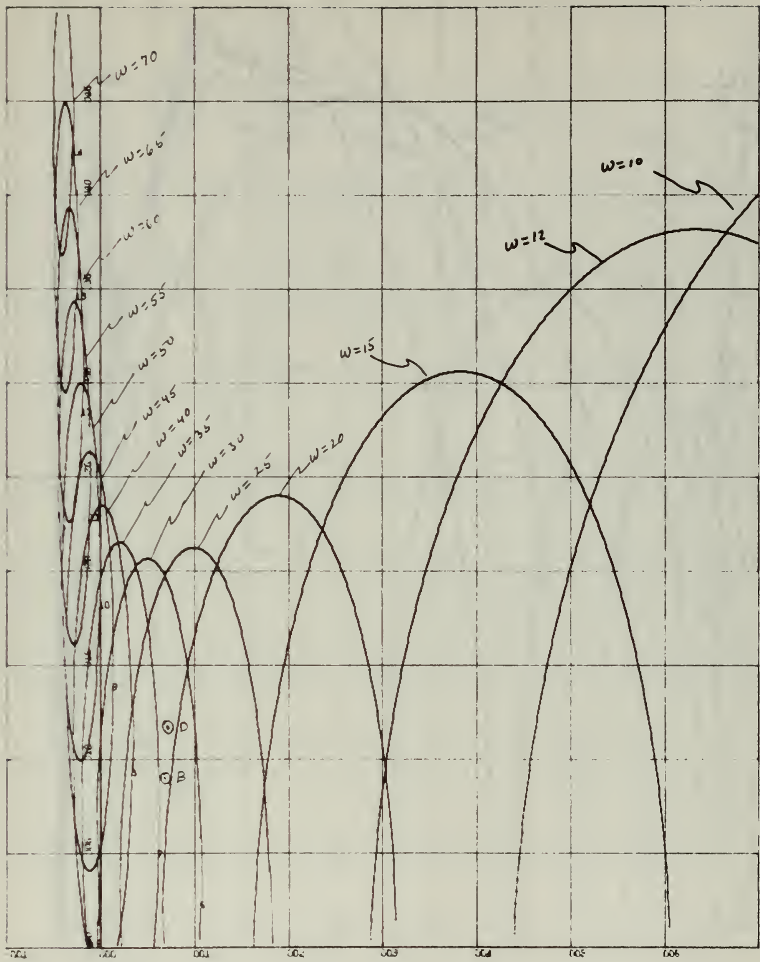
X-SCALE=4.00E+00 UNITS INCH.
Y-SCALE=4.00E-01 UNITS INCH.
GLAVIS, PARAM 7, 3RD ORDER T/A F.B., BETA CURVES
ORDINATE=PHASE, ABCISSA=OMEGA,CONALPHA=.00047

FIGURE 4-7



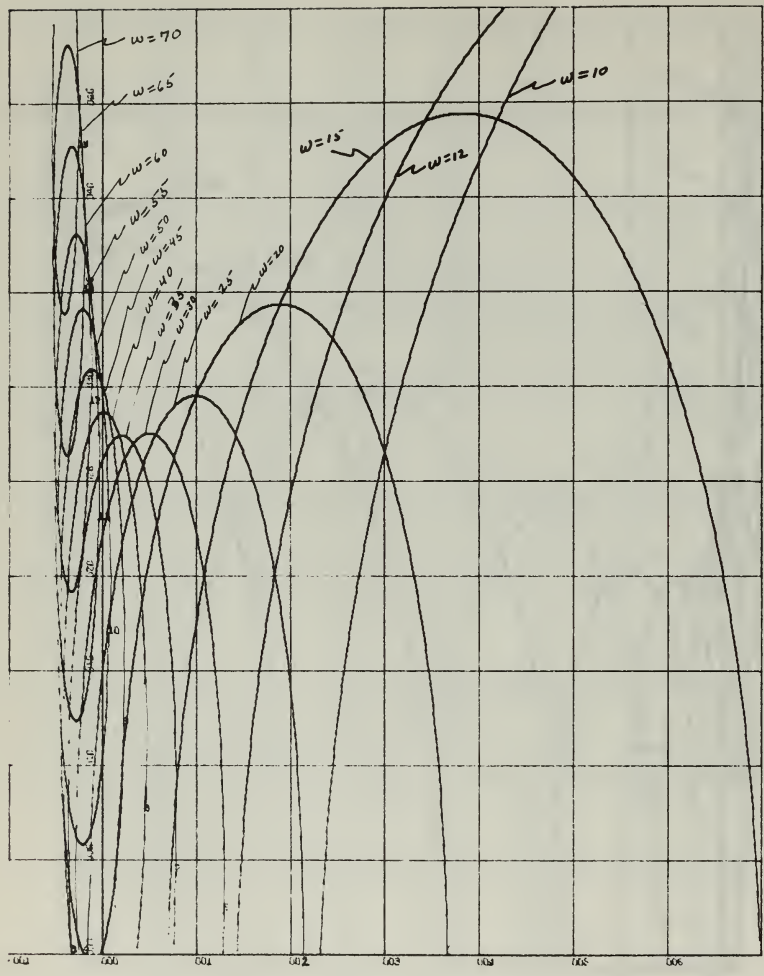
X-SCALE=1.00E-03 UNITS INCH.
Y-SCALE=5.00E-03 UNITS INCH.
GLAVIS, PARAM 1, 3RD ORDER T/A F.B.,OMEGA CURVES
ORDINATE=BETA, ABCISSA=ALPHA, CONMAGNITUDE=12DB

FIGURE 4-8



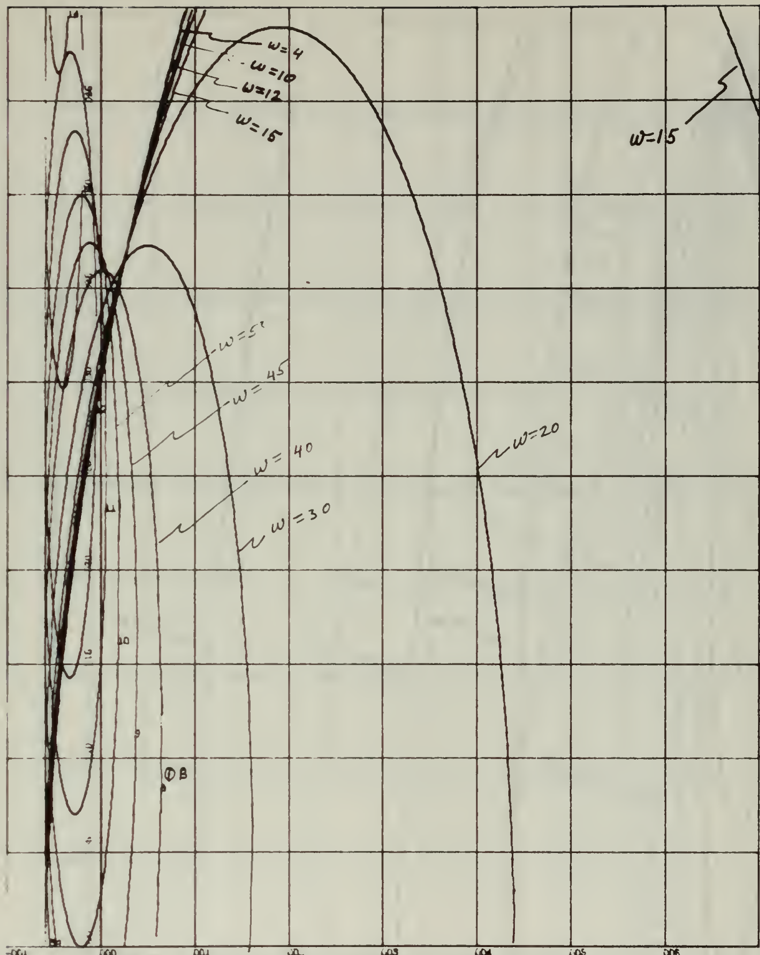
X-SCALE=1.00E-03 UNITS INCH.
Y-SCALE=5.00E-03 UNITS INCH.
GLAVIS, PARAM 1, 3RD ORDER T/A F.B.,OMEGA CURVES
ORDINATE=BETA, ABCISSA=ALPHA, CONMAGNITUDE=6DB

FIGURE 4-9



X-SCALE=1.00E-03 UNITS INCH.
Y-SCALE=5.00E-03 UNITS INCH.
GLAVIS, PARAM 1, 3RD ORDER T/A F.B., OMEGA CURVS
ORDINATE=BETA, ABCISSA=ALPHA, CONMAGNITUDE=3DB

FIGURE 4-10



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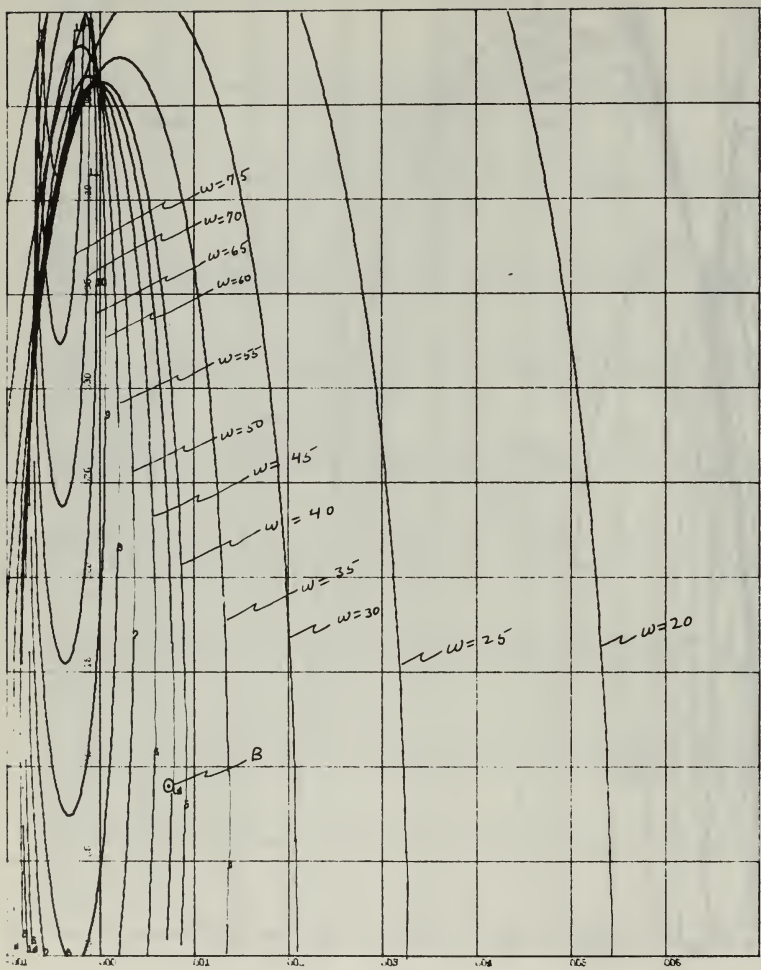
:LAVIS, PARAM 1, 3RD ORDER T/A F.B.,OMEGA CURVES
:RDINATE=BETA, ABCISSA=ALPHA, CONMAGNITUDE=0DB

FIGURE 4-11



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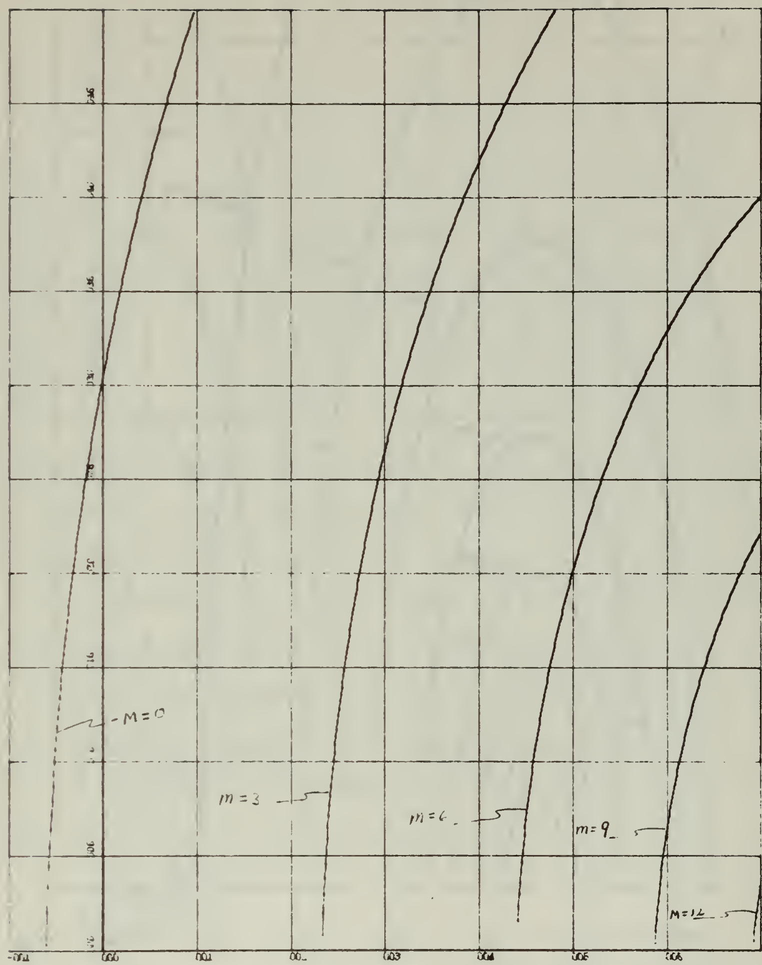
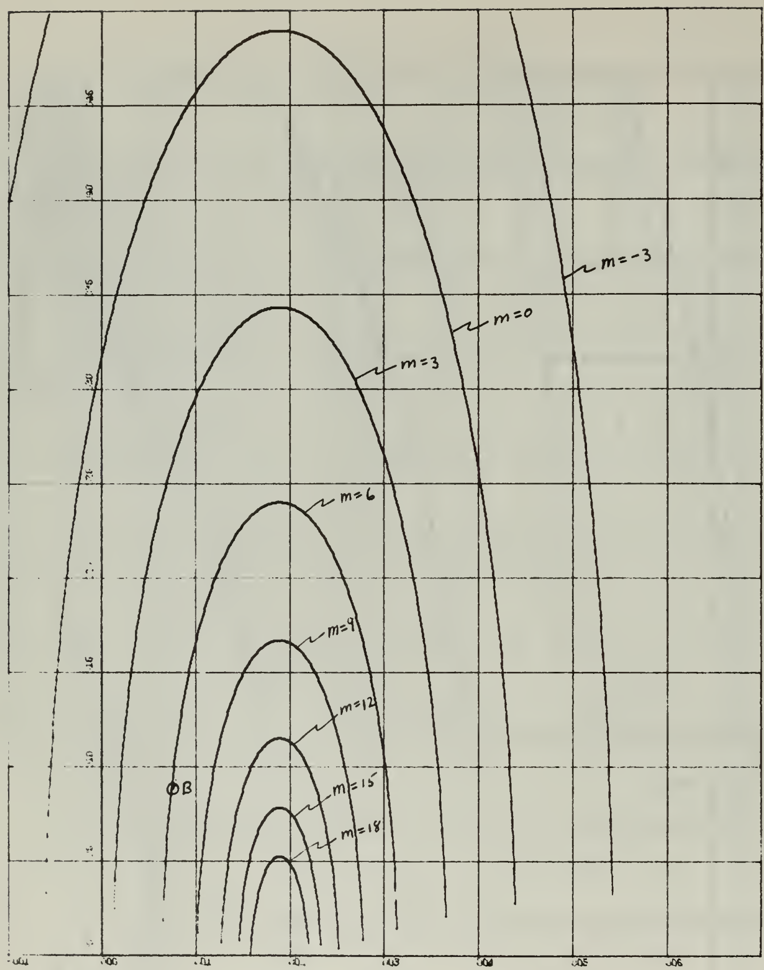
HUIS, PARAM 1, 3RD ORDER T A F.B.,OMEGA CURVES
RDINATE=BETA, ABCISSA=ALPHA, CONMAGNITUDE=-3DB

FIGURE 4-12



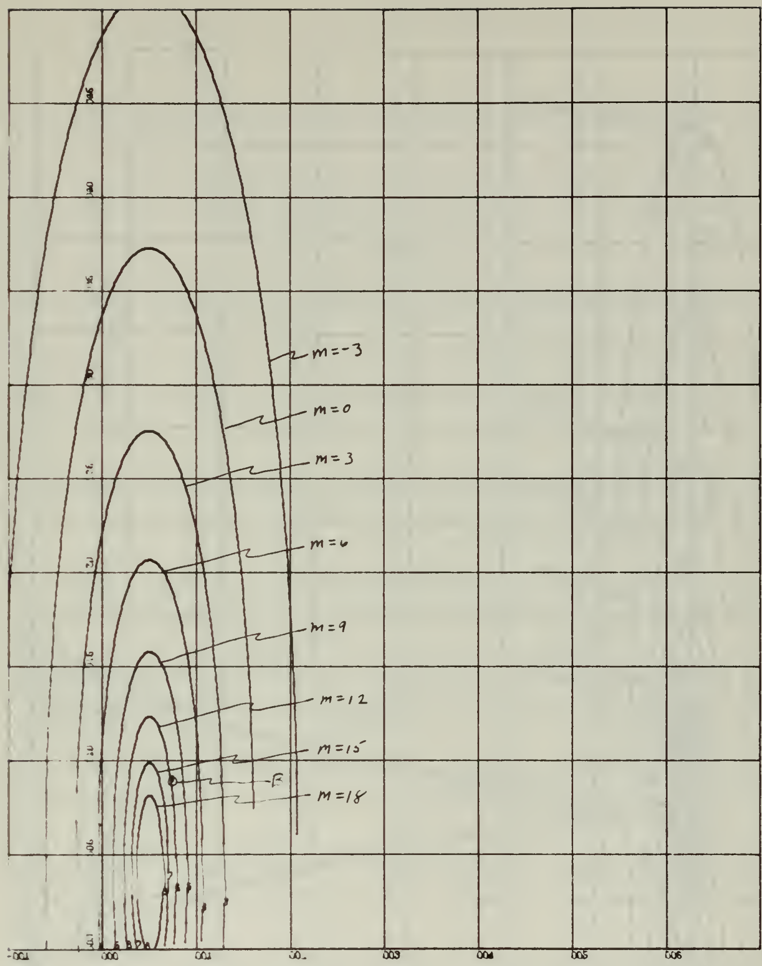
Y-SCALE=1.00E-03 UNITS INCH.
X-SCALE=5.00E-03 UNITS INCH.
ALAVIS, PARAM 2, 3RD ORDER T/R F.B., ~~OMEGA~~ MAGNITUDE CURVES
ORDINATE=BETA, ABCISSA=ALPHA, CONOMEGA=10RADIANS

FIGURE 4-13



X-SCALE=1.00E-03 UNITS INCH.
Y-SCALE=5.00E-03 UNITS INCH.
DAVIS, PARAM 2, 3RD ORDER T & F.B., MAGNITUDE ~~OMEGA~~ CURVES
ORDINATE=BETA, ABCISSA=ALPHA, CONOMEGA=20RADIANS

FIGURE 4-14



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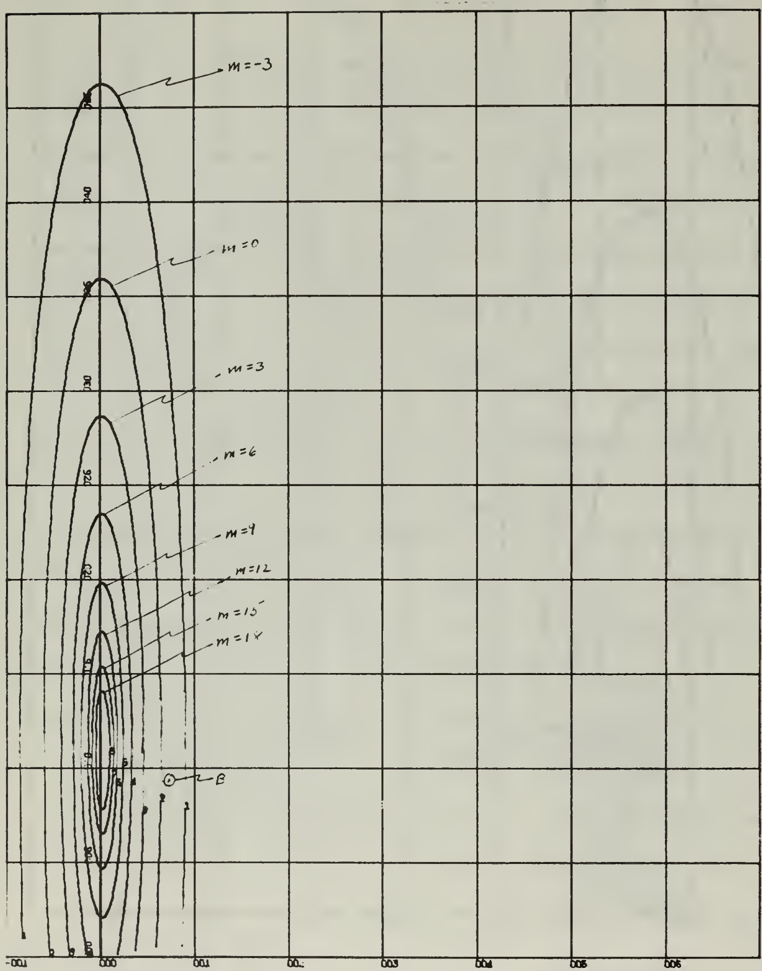
Y-SCALE=5.00E-03 UNITS INCH.
MAGNITUDE
SLAVIS, PARAM 2, 3RD ORDER T/A F.B., ~~OMEGA~~ CURVES
ORDINATE=BETA, ABCISSA=ALPHA, CONOMEGA=30RADIANS

FIGURE 4-15



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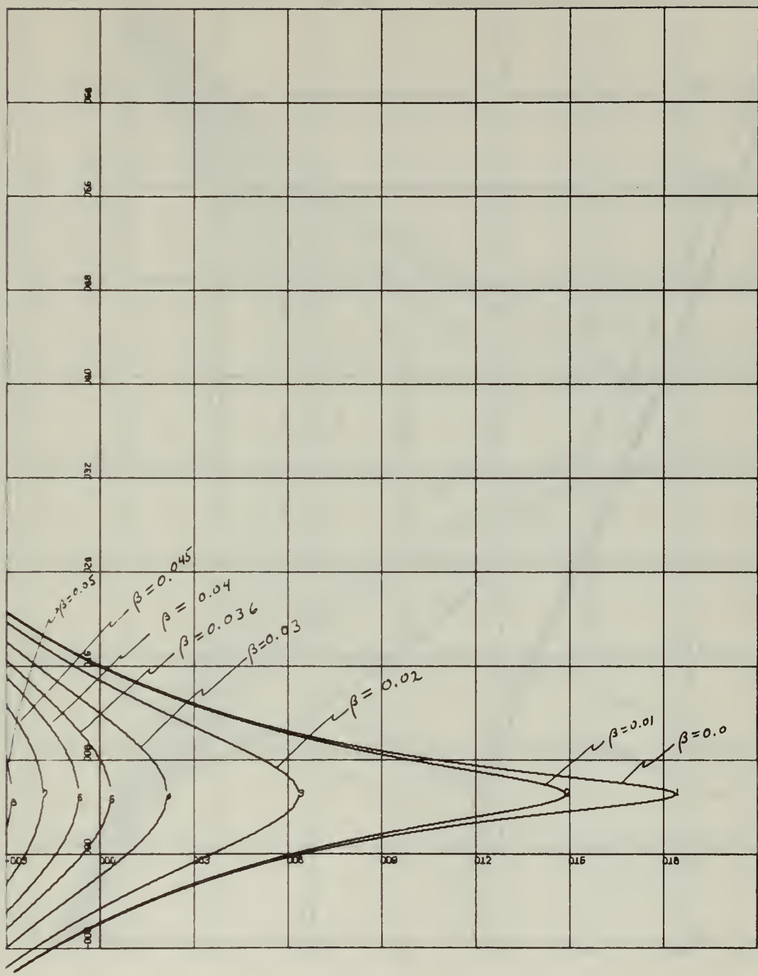
GLAVIS, PARAM 2, 3RD ORDER T/A F.B., MAGNITUDE ~~OMEGA~~ CURVES
ORDINATE=BETA, ABCISSA=ALPHA, CONOMEGA=40RADIANS

FIGURE 4-16



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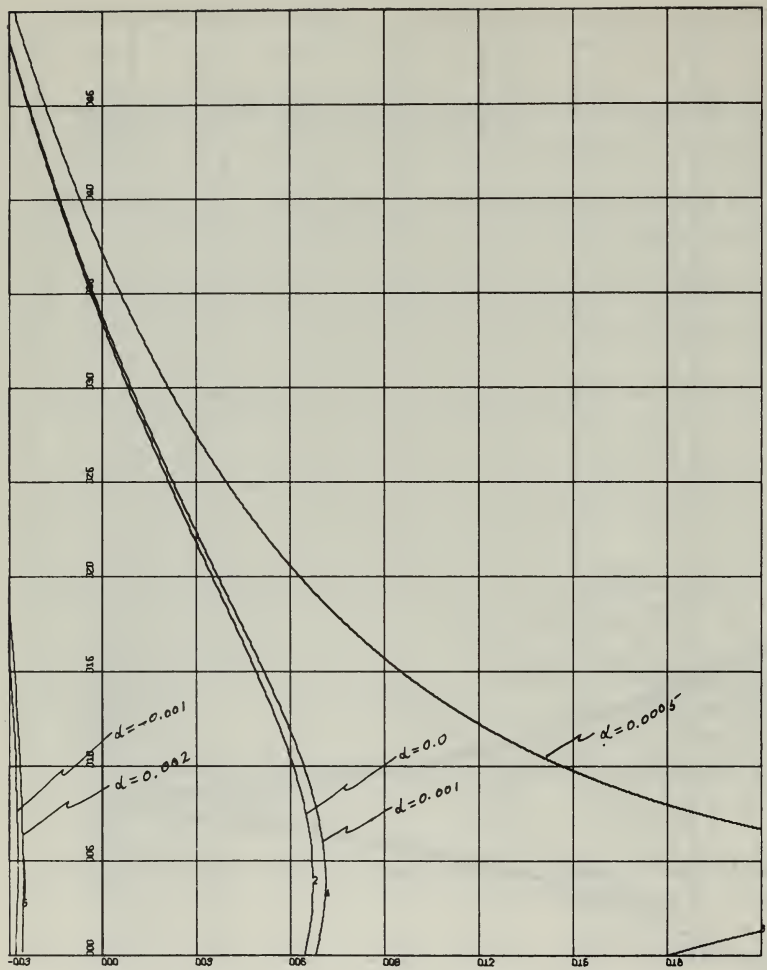
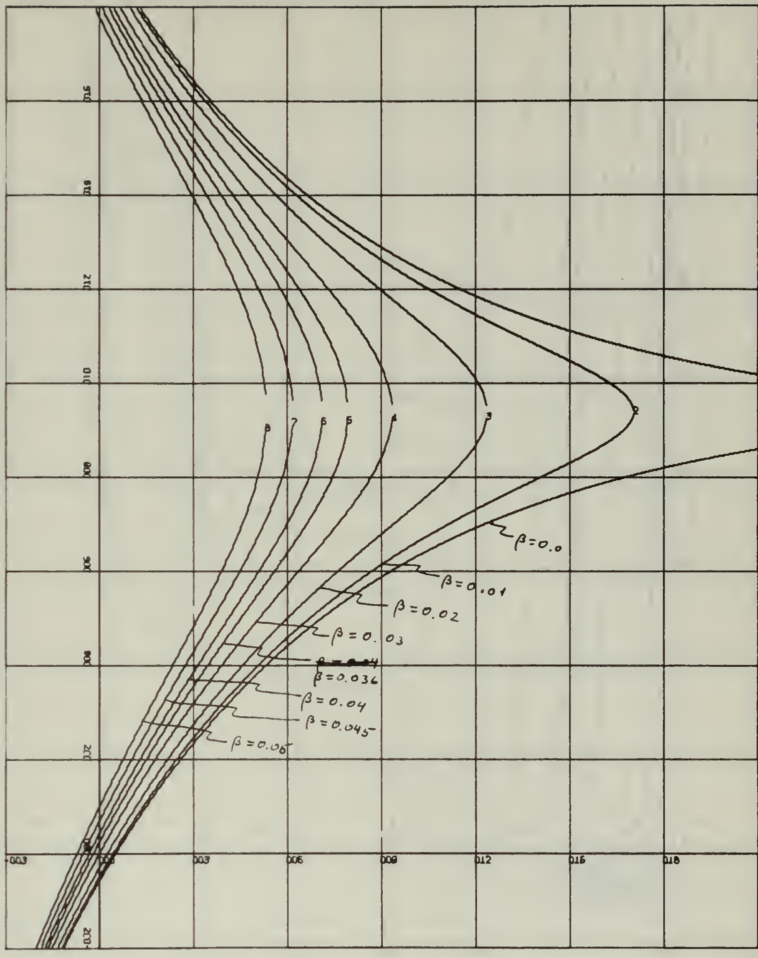
GLAVIS, PARAM 3, 3RD ØRDER T/A F.B., BETA CURVES.
ØRDINATE=ALPHA, ABCISSA=MAG , CØNØMEGA=30RAD/SEC

FIGURE 4-17



X-SCALE=3.00E+00 UNITS INCH.
Y-SCALE=5.00E-03 UNITS INCH.
ORDINATE=BETA , ABCISSA=MAG , CONOMEGA=30RAD/SEC
GLAVIS, PARAM 3, 3RD ORDER T/A F.B.,ALPHA CURVES

FIGURE 4-18



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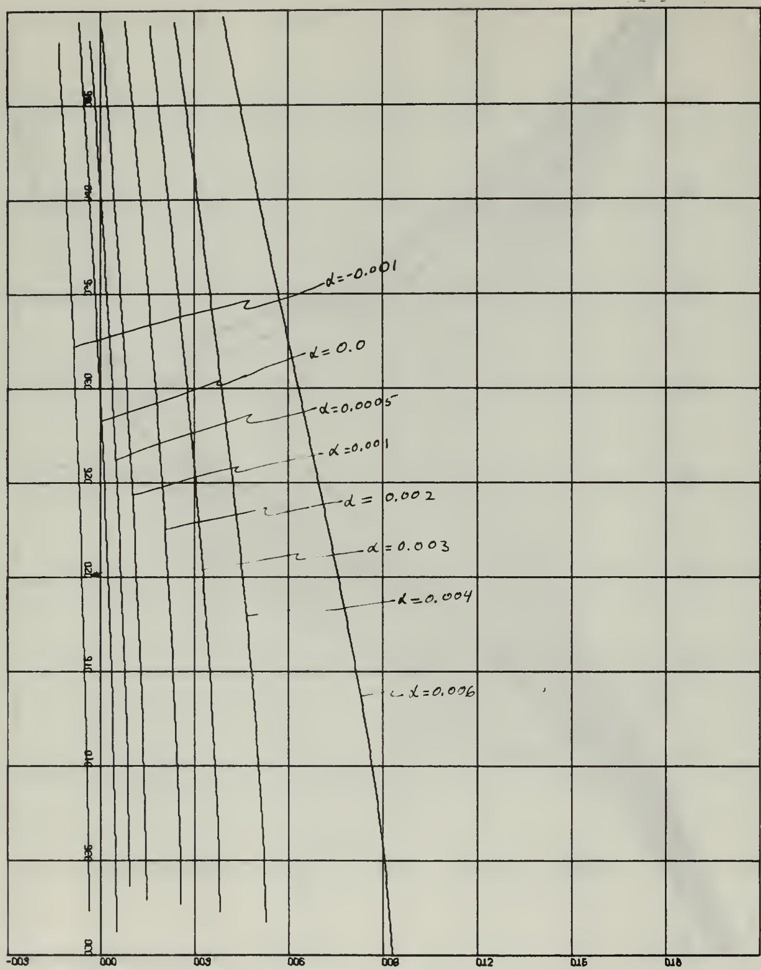
Y-SCALE=2.00E-03 UNITS INCH.
GLAVIS, PARAM 3, 3RD ORDER T/A F.B., BETA CURVES
ORDINATE=ALPHA, ABCISSA=MAG , CONOMEGA=10RAD/SEC

FIGURE 4-19



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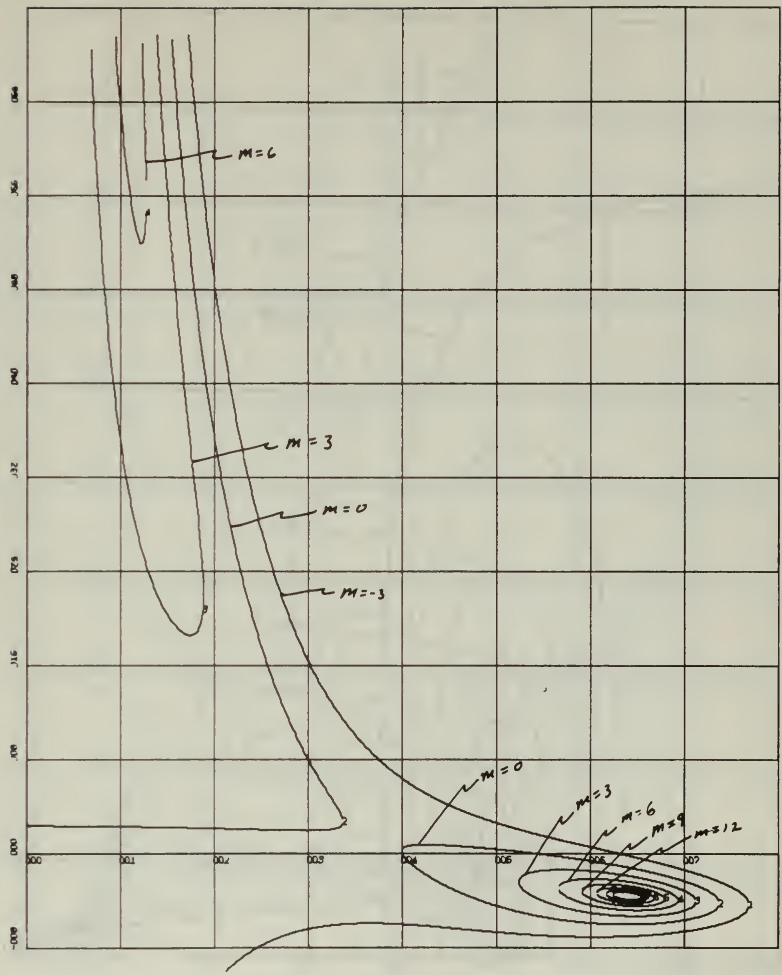
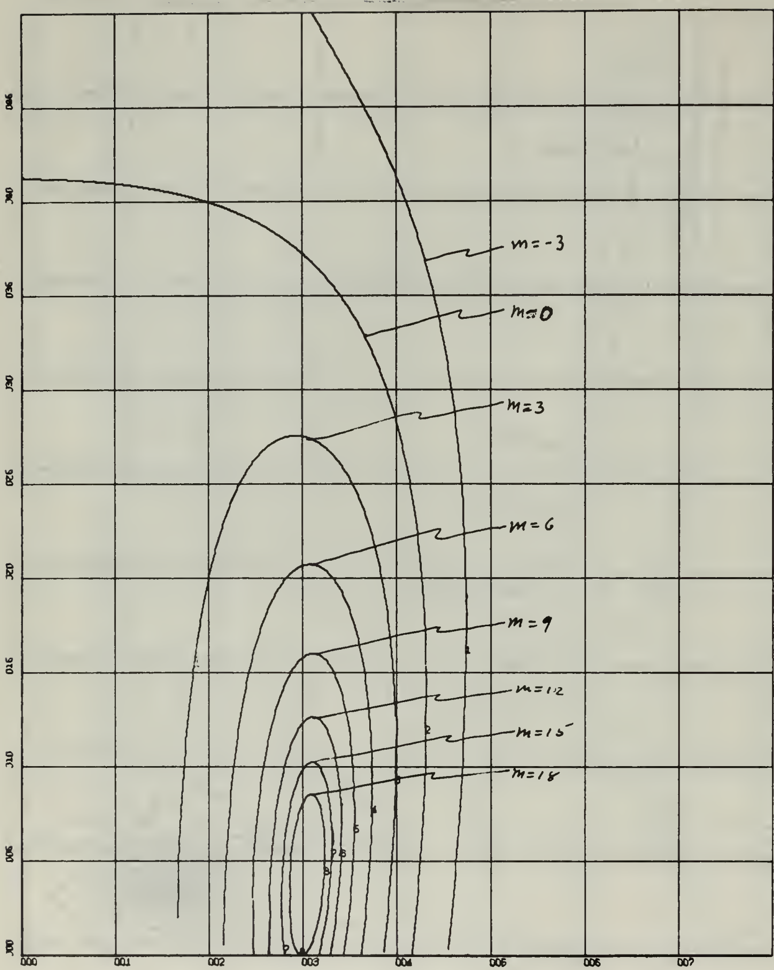
Y-SCALE=5.00E-03 UNITS INCH.
GLAVIS, PARAM 3, 3RD ORDER T/A F.B.,ALPHA CURVES
ORDINATE=BETA , ABCISSA=MAG , CONOMEGA=10RAD/SEC

FIGURE 4-20



X-SCALE=1.00E+01 UNITS INCH.
Y-SCALE=8.00E-04 UNITS INCH.
GLAVIS, PARAM 4, 3RD ORDER T/A F.B., MAG CURVES
ORDINATE=ALPHA, ABCISSA=OMEGA, CONBETA=0.036

FIGURE 4-21



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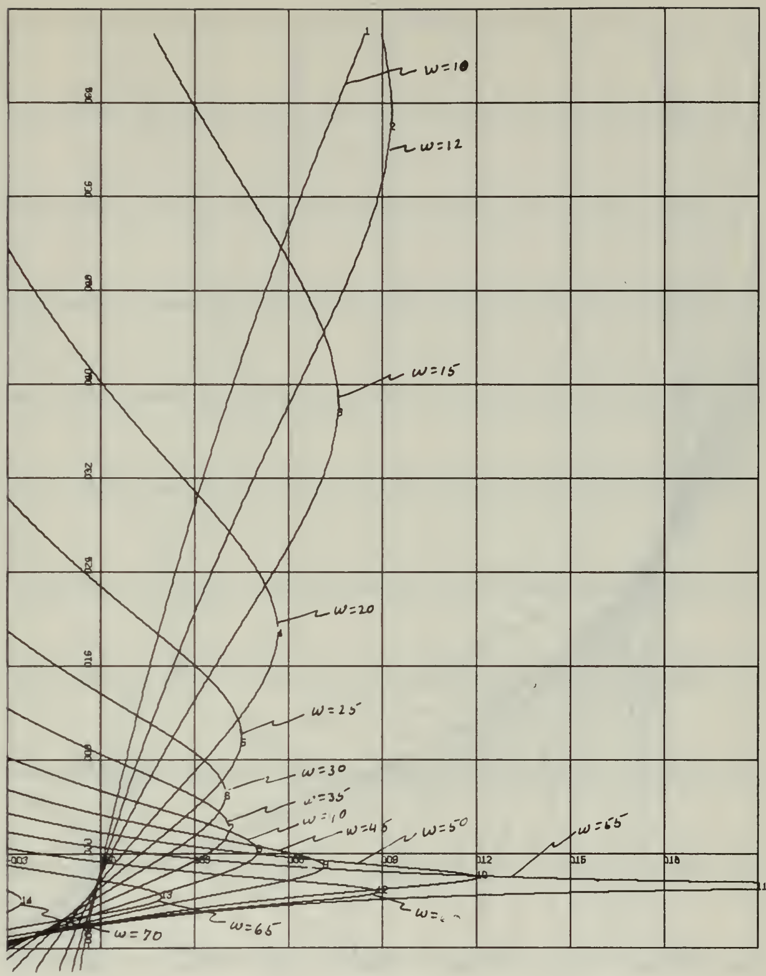
Y-SCALE=5.00E-03 UNITS INCH.
GLAVIS, PARAM 4, 3RD ØRDER T/A F.B., MAG CURVES
ØRDINATE=BETA , ABCISSA=ØMEGA, CØNALPHA=0.00047

FIGURE 4-22



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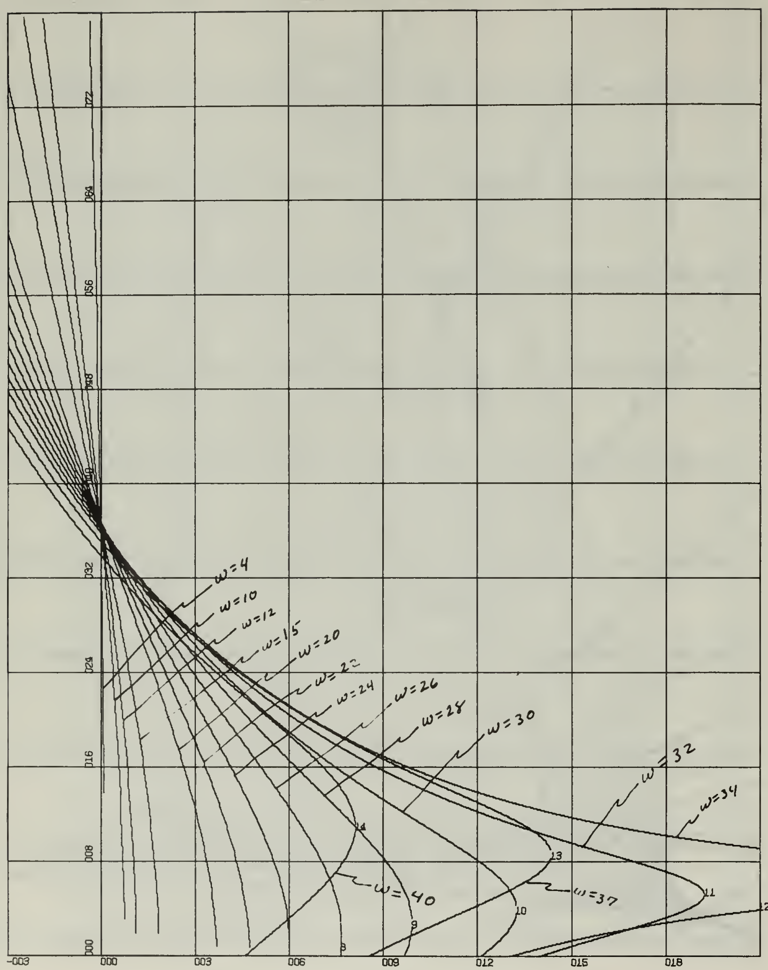
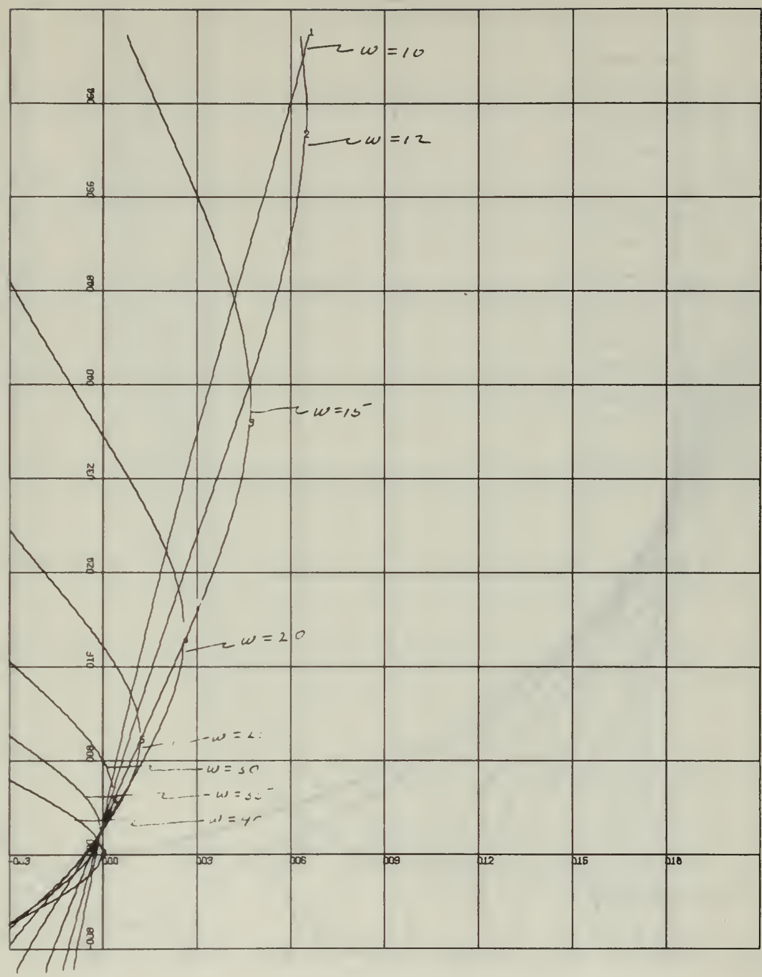
Y-SCALE=8.00E-04 UNITS INCH.
GLAVIS, PARAM 5, 3RD ØRDER T/A F.B.,ØMEGA CURVES
ØRDINATE=ALPHA, ABCISSA=MAG , CØNBETA=0.025

FIGURE 4-23



X-SCALE=3.00E+00 UNITS INCH.
Y-SCALE=8.00E-03 UNITS INCH.
GLAVIS, PARAM 5, 3RD ØRDER T/A F.B.,ØMEGA CURVES
ØRDINATE=BETA , ABCISSA=MAG , CØNALPHA=0.00027

FIGURE 4-24



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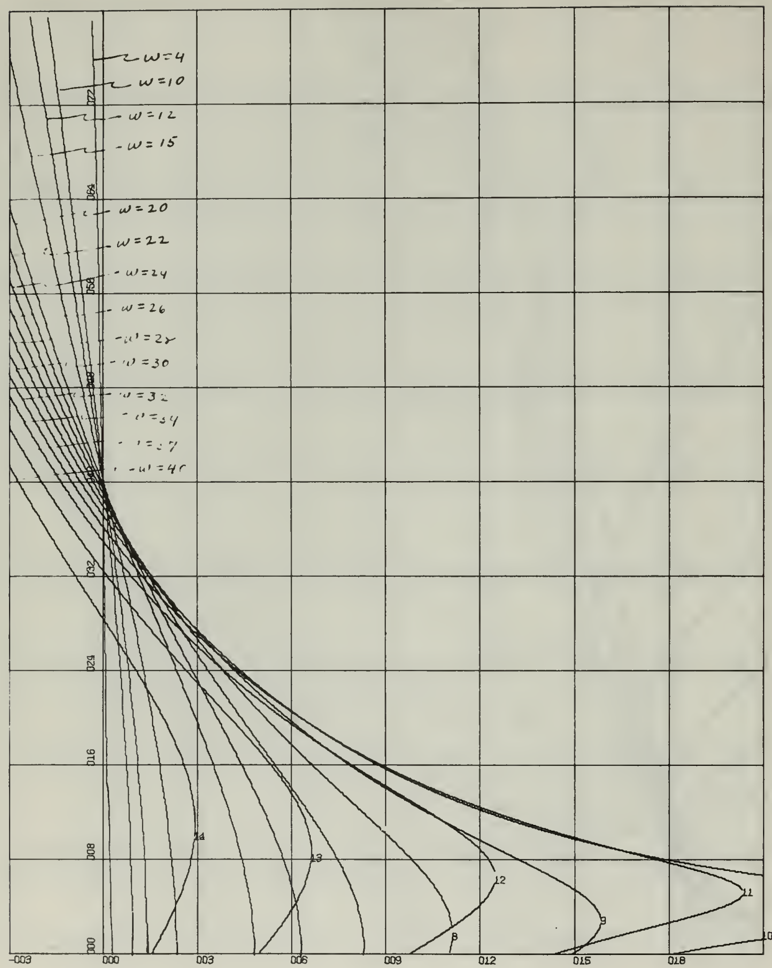
Y-SCALE=8.00E-04 UNITS INCH.
GLAVIS, PARAM 5, 3RD ØRDER T/A F.B.,ØMEGA CURVES
ØRDINATE=ALPHA, ABCISSA=MAG , CØNBETA=0.036

FIGURE 4-25



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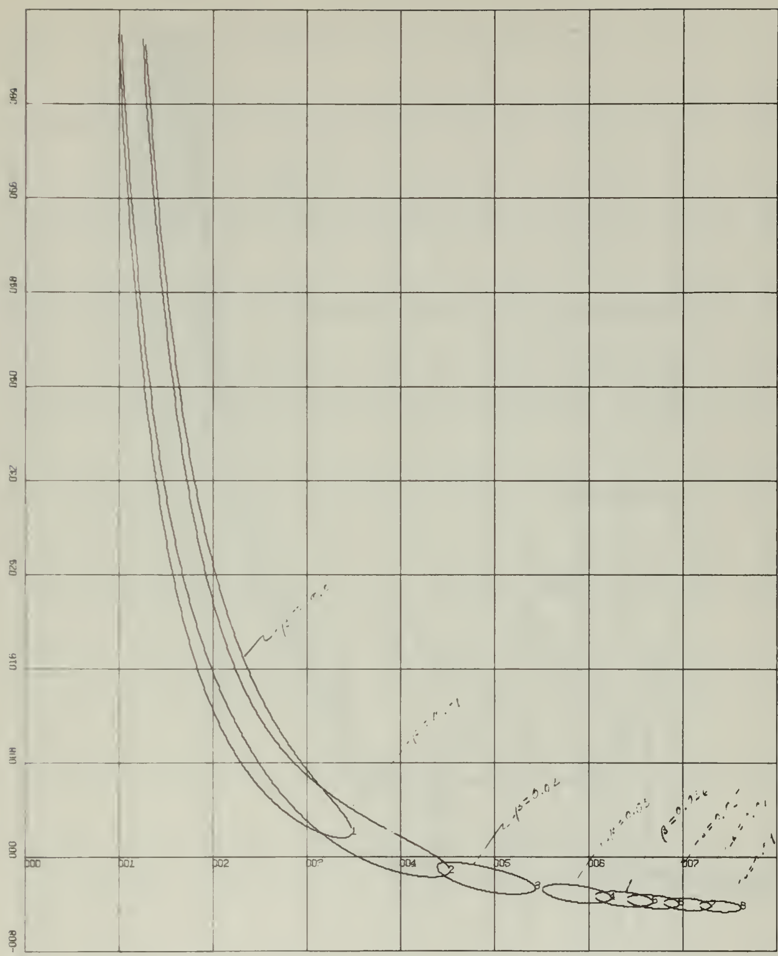
Y-SCALE=8.00E-03 UNITS INCH.
GLAVIS, PARAM 5, 3RD ØRDER T/A F.B.,ØMEGA CURVES
ØRDINATE=BETA , ABCISSA=MAG , CØNALPHA=0.00047

FIGURE 4-26



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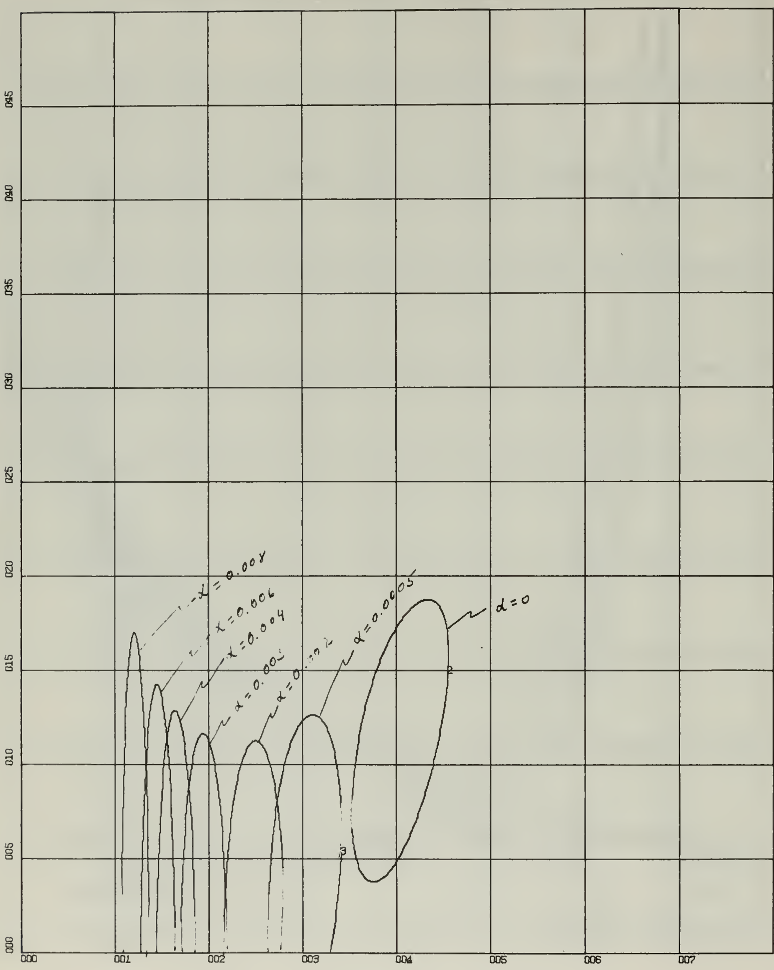
Y-SCALE=8.00E-04 UNITS INCH.
GLAVIS, PARAM 6, 3RD ØRDER T/A F.B., BETA CURVES
ØRDINATE=ALPHA, ABCISSA=ØMEGA, CØNMAGNITUDE=12DB

FIGURE 4-27



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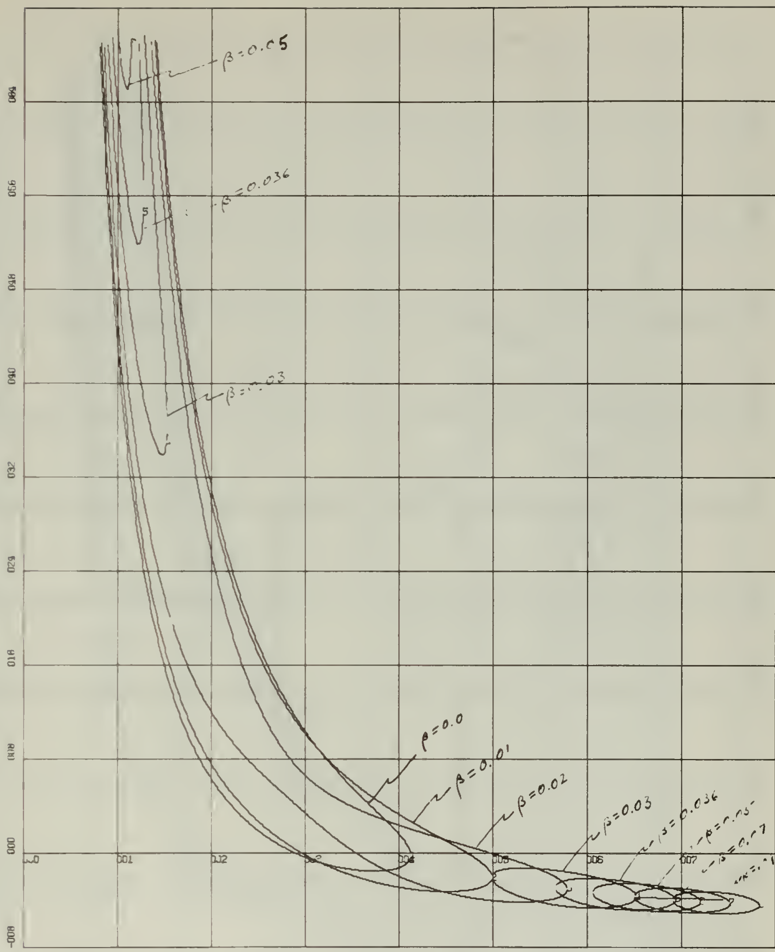
GLAVIS, PARAM 6, 3RD ØRDER T/A F.B.,ALPHA CURVES
ØRDINATE=BETA , ABCISSA=ØMEGA, CØNMAGNITUDE=12DB

FIGURE 4-28



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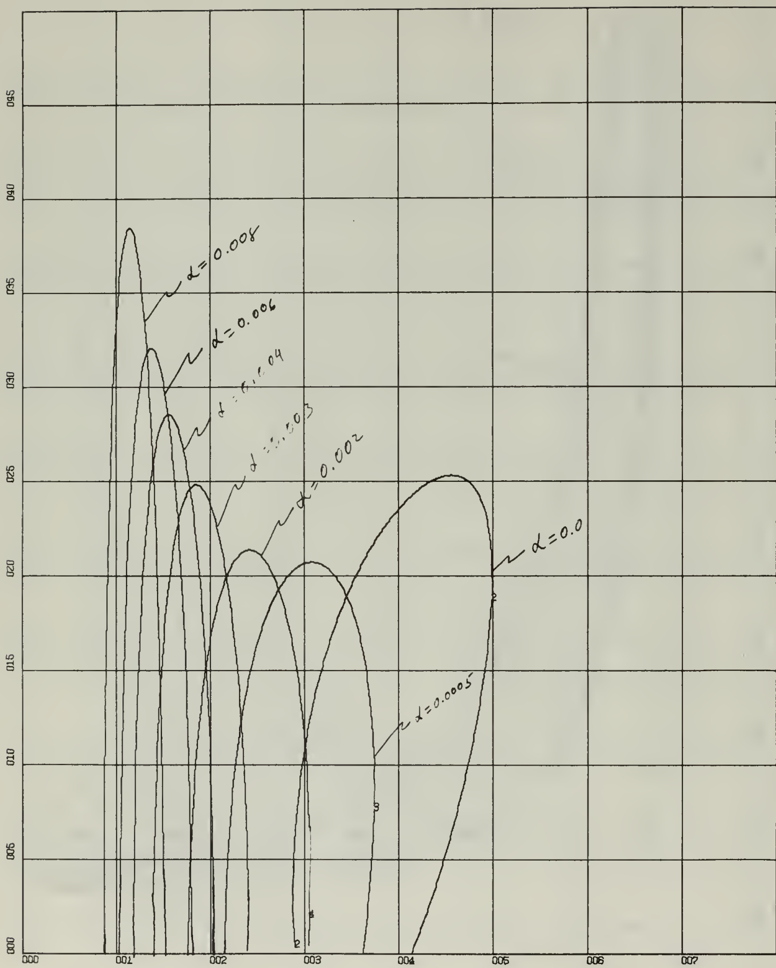
Y-SCALE=8.00E-04 UNITS INCH.
GLAVIS, PARAM 6, 3RD ØRDER T/A F.B., BETA CURVES
ØRDINATE=ALPHA, ABCISSA=ØMEGA, CØNMAGNITUDE=6DB

FIGURE 4-29



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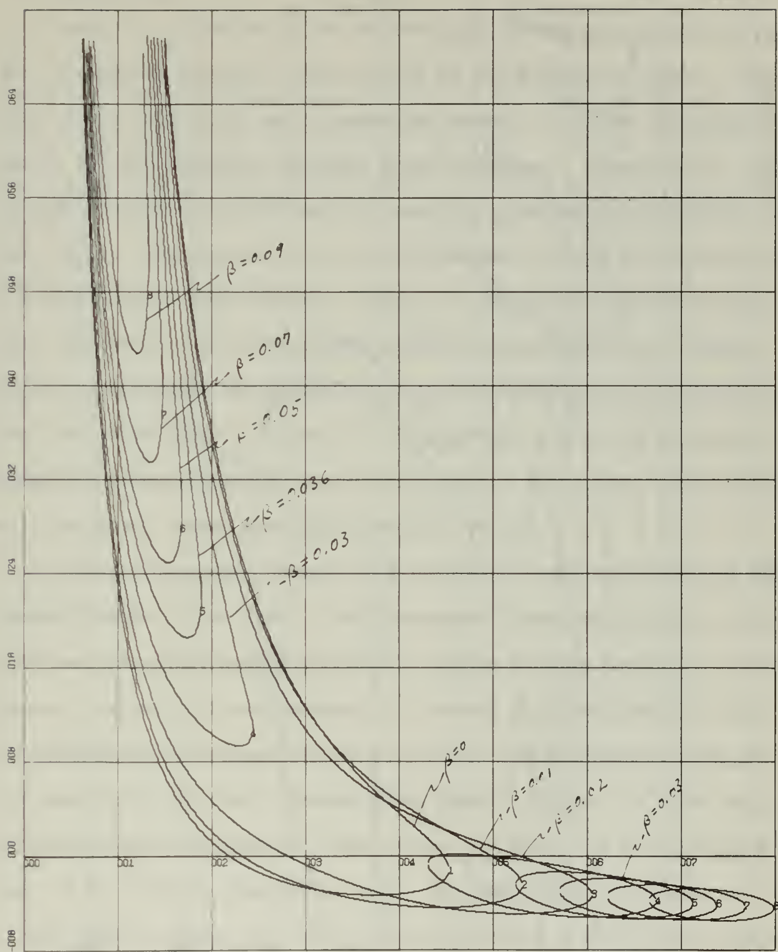
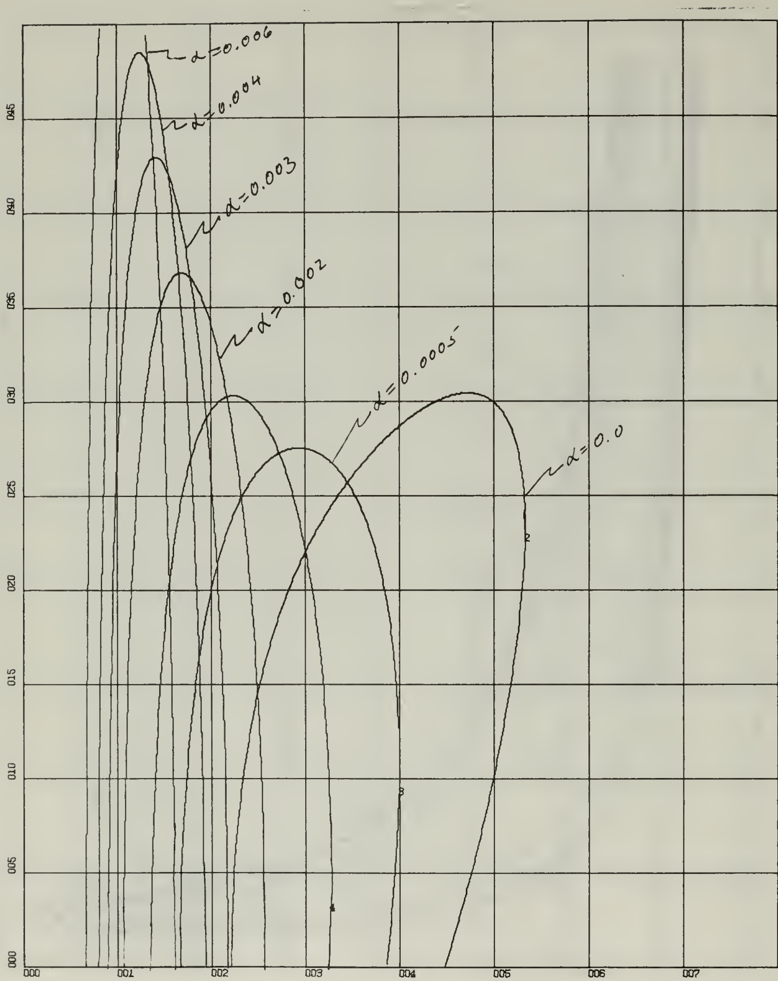
Y-SCALE=5.00E-03 UNITS INCH.
GLAVIS, PARAM 6, 3RD ØRDER T/A F.B.,ALPHA CURVES
ØRDINATE=BETA , ABCISSA=ØMEGA, CØNMAGNITUDE=6DB

FIGURE 4-30



X-SCALE=1.00E+01 UNITS INCH.
Y-SCALE=8.00E-04 UNITS INCH.
GLAVIS, PARAM 6, 3RD ØRDER T/A F.B., BETA CURVES
ØRDINATE=ALPHA, ABCISSA=ØMEGA, CØNMAGNITUDE=3DB

FIGURE 4-31



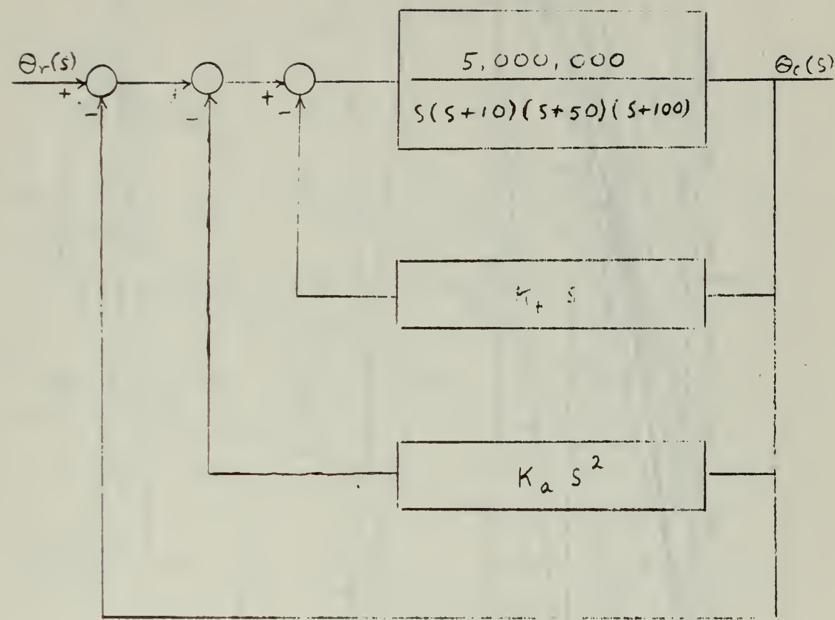
X-SCALE=1.00E+01 UNITS INCH.
Y-SCALE=5.00E-03 UNITS INCH.
GLAVIS, PARAM 6, 3RD ORDER T/A F.B.,ALPHA CURVES
ORDINATE=BETA , ABCISSA=OMEGA, CONMAGNITUDE=3DB

FIGURE 4-32

5. CONCLUSIONS

Section 4 presented an exhaustive study of the application of frequency response parameter plane curves to one particular system. The selection of the third order system was based on the ease of interpretation of each frequency response graph discussed. Extensions to systems of higher order will undoubtedly change the graphical configurations to some extent. The degree of this variation was smaller than expected for a similar fourth order system. Figure 5-1 shows the particular fourth order tachometer and acceleration feedback system which was studied. Figure 5-2 presents the system stability performance curves in the alpha-beta parameter plane. Figures 5-3 through 5-12 depict the associated frequency response curves. Comparison with the third order system discussed in section 4 shows strikingly similar results.

Several concluding comments are noteworthy concerning the use PARAM-1 through PARAM-6. The alpha - beta parameter plane provides more information than any other representation for system studies involving the simultaneous variation of two parameters. Several different PARAM-1 plots are considered to be most useful for initial system studies, when they are accompanied by their corresponding PARAM-6 graphs. In this manner, a three dimensional space of alpha, beta, and omega can be visualized for each of the constant magnitudes used. The additional incorporation of one or two PARAM-2 graphs will describe the effect of a variation in the fourth dimension. Stated in other words, the information in PARAM-2 shows how the three dimensional PARAM-1, PARAM-6 surface varies in magnitude. PARAM-3 and PARAM-5 curves yield a more sensitive portrayal of the magnitude variations; however these are restricted to a single parameter variation. They are useful for accurately presenting a maximum peak resonance magnitude when this information is not available from the peak resonant tangent locus of PARAM-1. PARAM-4 effectively shows a series of Bode curves as a function of one system parameter. As such they would be advantageous for intermediate studies, when the final adjustments are being contemplated. Finally, the familiar Bode plot representations of PARAM-7 show the most accurate frequency response, and are therefore most useful in the terminal stages of design.

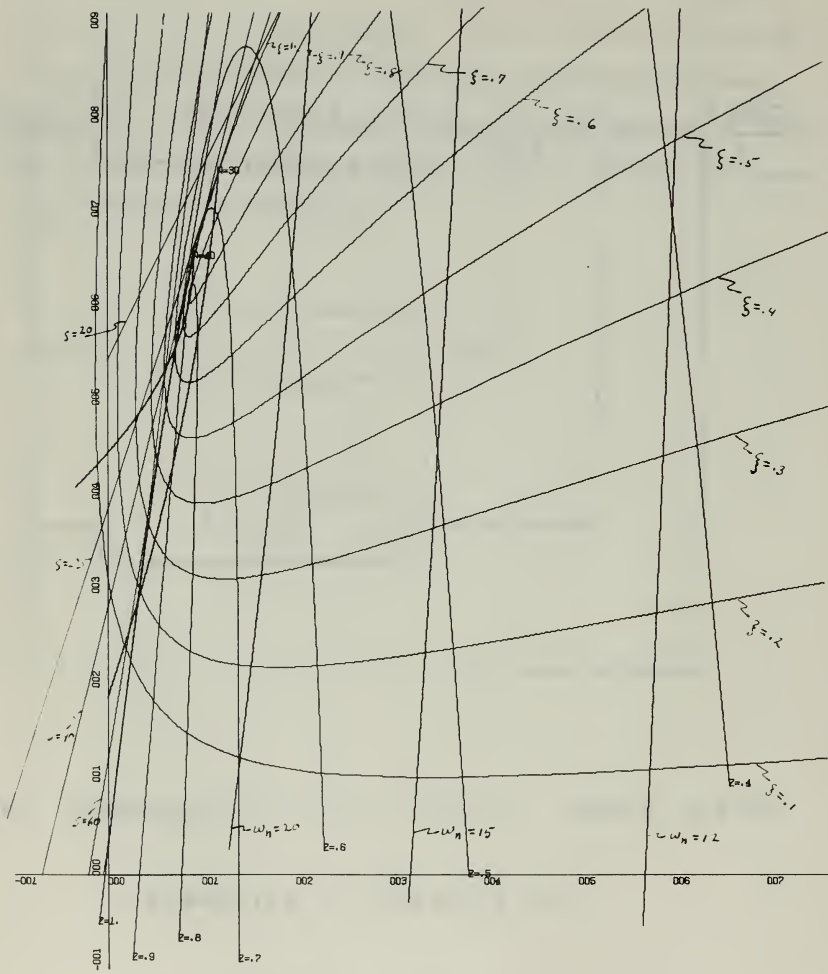


FOURTH ORDER SYSTEM WITH TACHOMETER AND ACCELERATION FEEDBACK

CHARACTERISTIC EQUATION

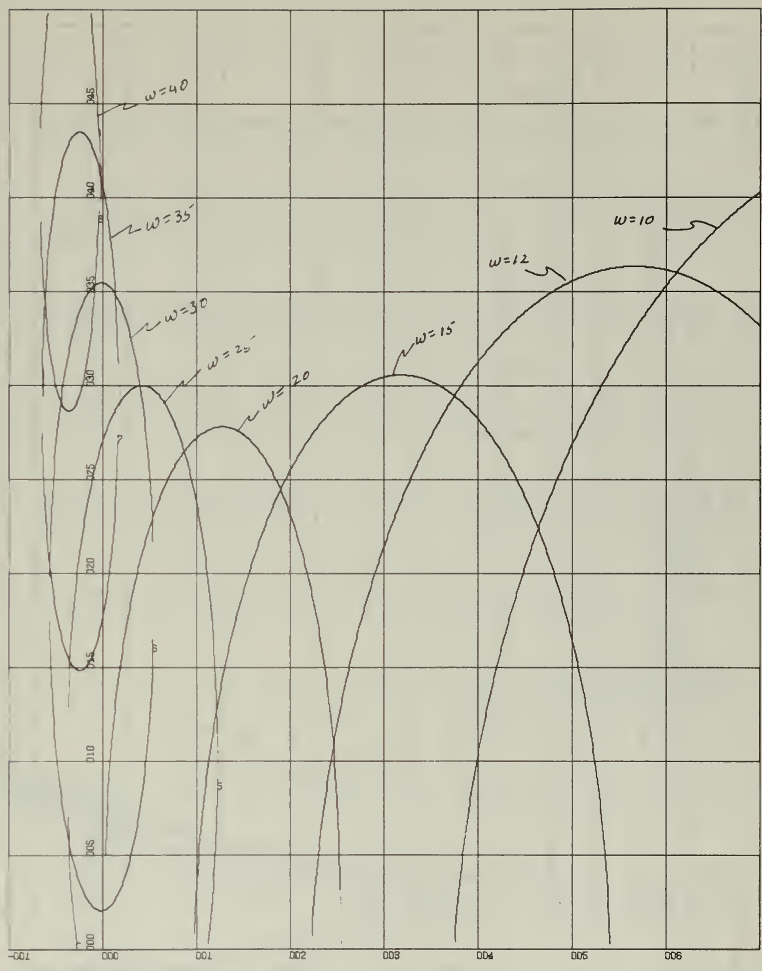
$$S^4 + 160S^3 + (6500 + 5{,}000{,}000K_a)\,S^2 + (50{,}000 + 5{,}000{,}000\,K_t)\,S + 5{,}000{,}000 = 0$$

FIGURE 5-1



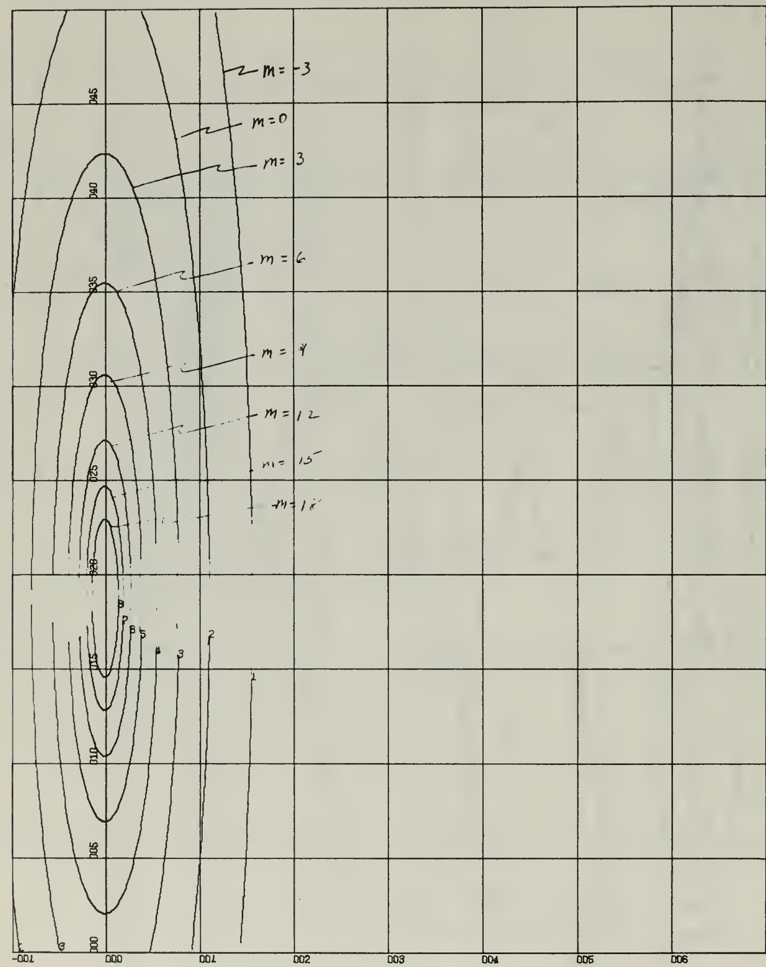
X-SCALE=1.00E-03 UNITS INCH.
Y-SCALE=1.00E-02 UNITS INCH.
GLAVIS, PARAMETER PLANE CURVES, 4TH ORDER T/A FB
S**4+160S**3+(6.5E3+5E6A)S**2+(5E4+5E6B)S+5E6=0

FIGURE 5-2



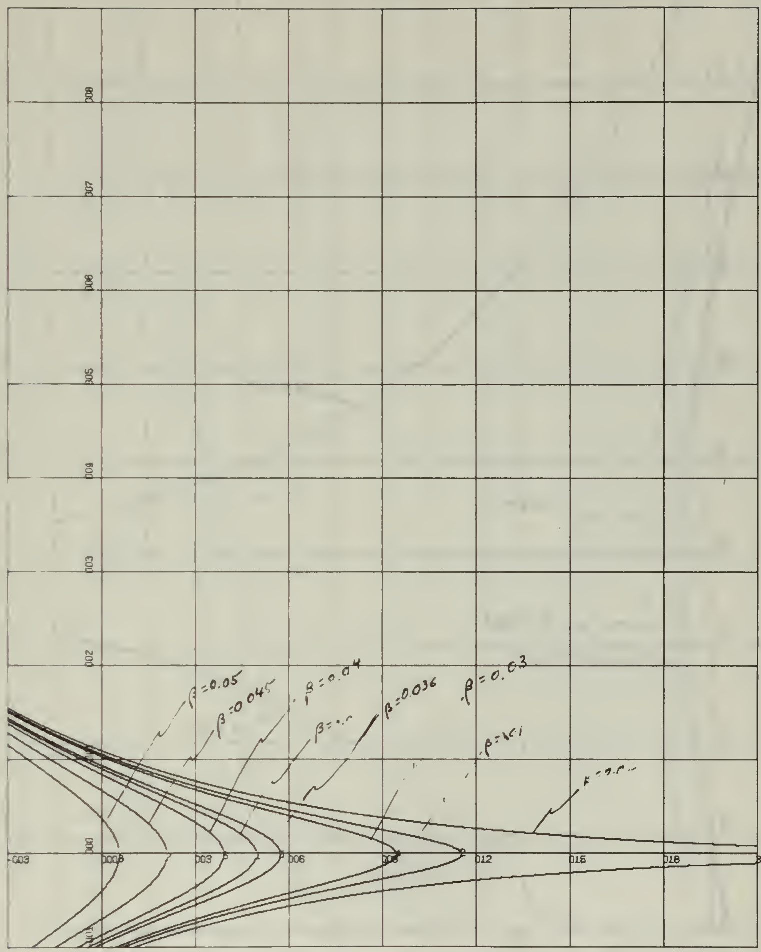
X-SCALE=1.00E-03 UNITS INCH.
Y-SCALE=5.00E-03 UNITS INCH.
GLAVIS, PARAM 1, 4TH ORDER T/A F.B.,OMEGA CURVES
ORDINATE=BETA, ABCISSA=ALPHA, CONMAGNITUDE= 6DB

FIGURE 5-3



X-SCALE=1.00E-03 UNITS INCH.
Y-SCALE=5.00E-03 UNITS INCH.
GLAVIS, PARAM 2, 4TH ORDER T/A F.B., MAG CURVES
ORDINATE=BETA, ABCISSA=ALPHA, CONOMEGA=30DB

FIGURE 5-4



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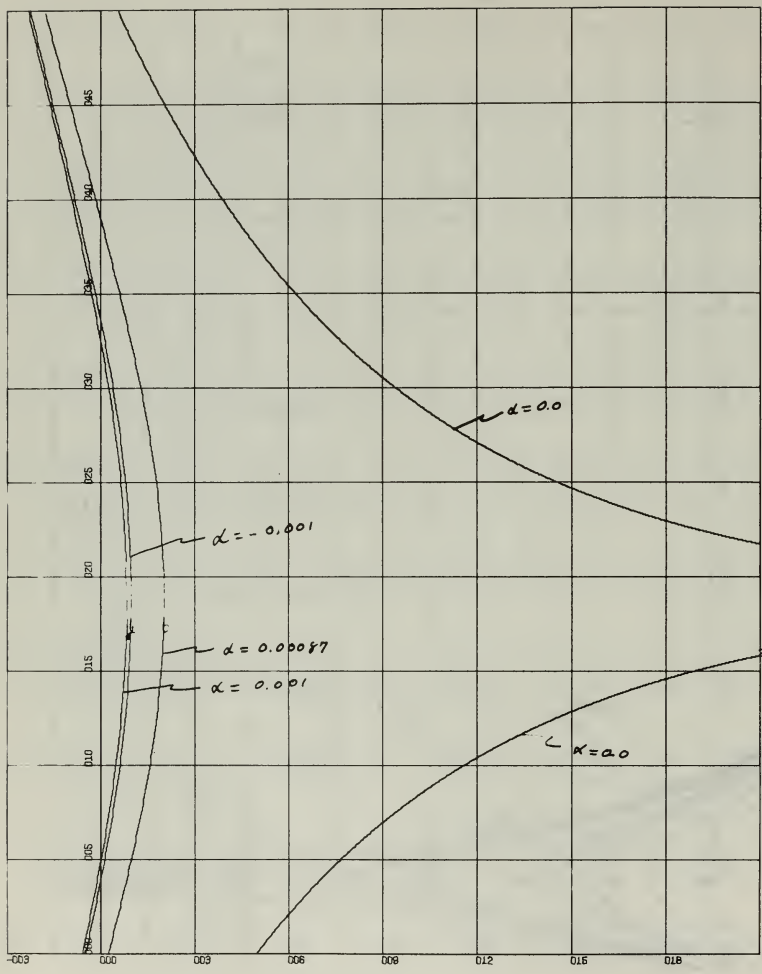
Y-SCALE=1.00E-03 UNITS INCH.
GLAVIS, PARAM 3, 4TH ØRDER T/A F.B., BETA CURVES
ØRDINATE=ALPHA, ABCISSA=MAG , CØNØMEGA=30RAD/SEC

FIGURE 5-5



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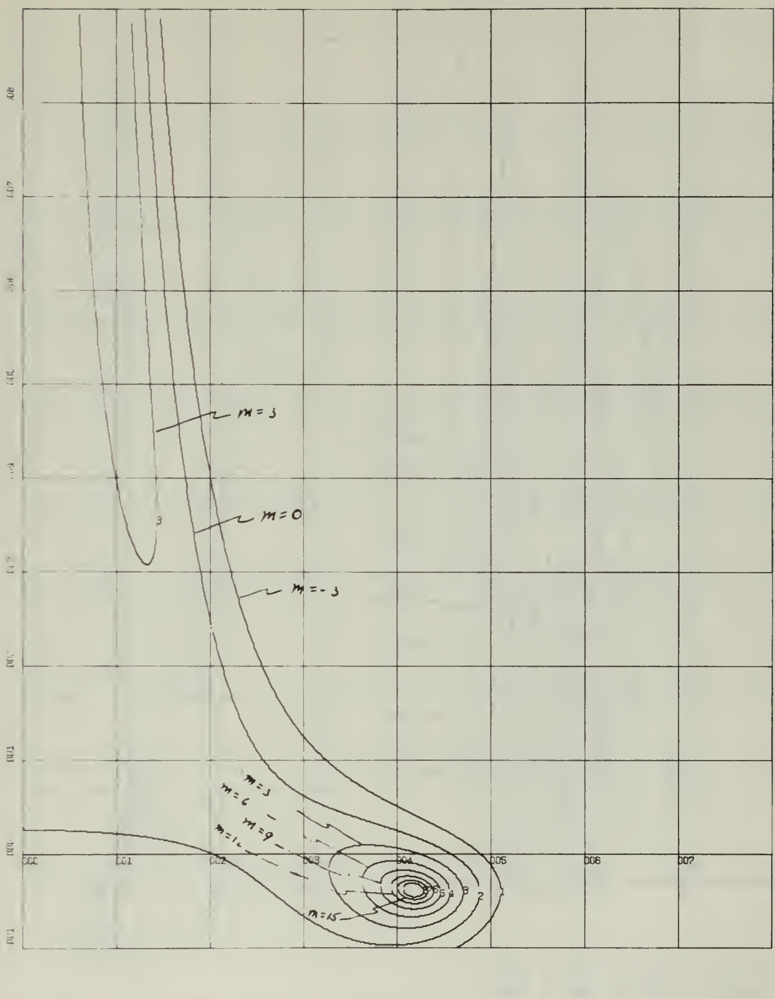
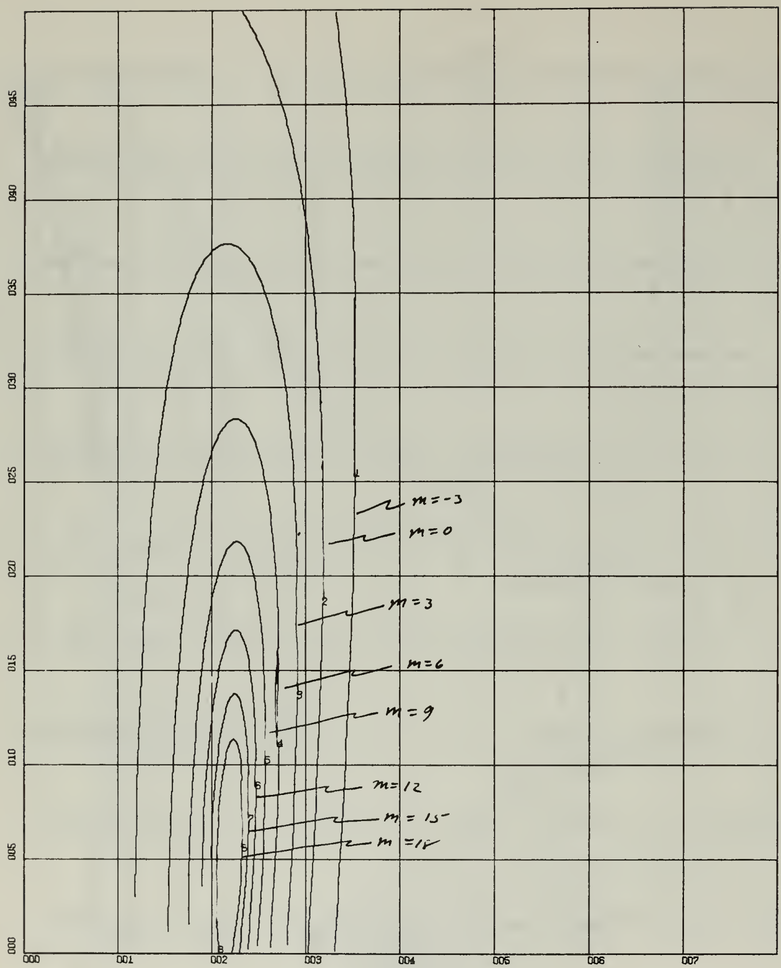
Y-SCALE=5.00E-03 UNITS INCH.
GLAVIS, PARAM 3, 4TH ORDER T/A F.B.,ALPHA CURVES
ORDINATE=BETA , ABCISSA=MAG , CONOMEGA=30RAD/SEC

FIGURE 5-6



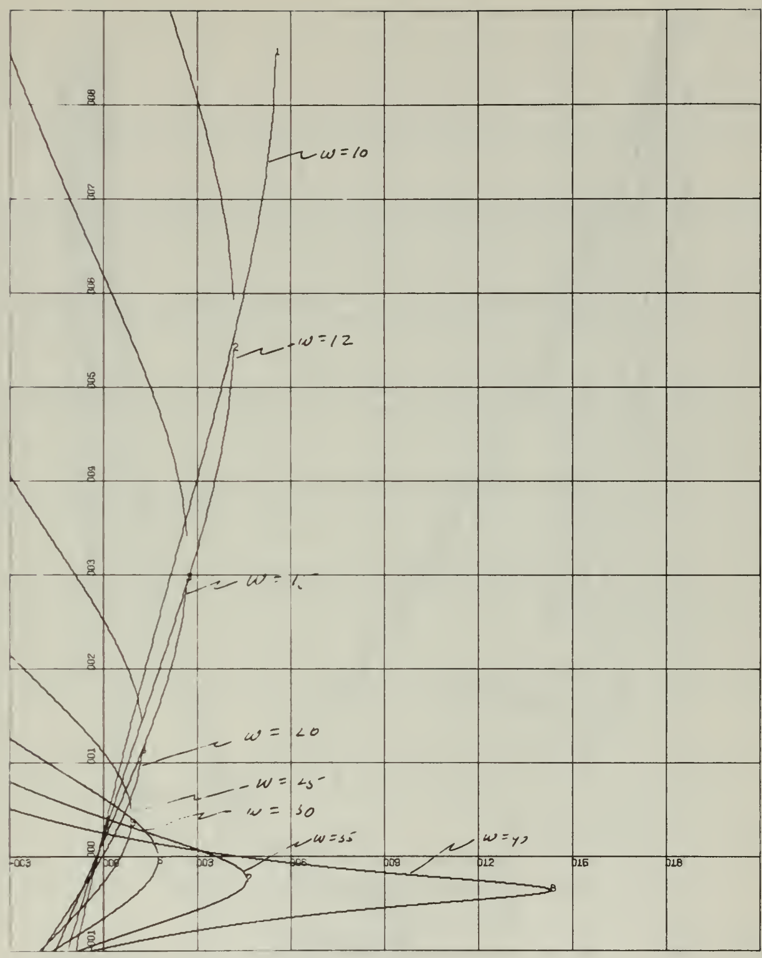
/-SCALE=1.00E+01 UNITS INCH.
/-SCALE=1.00E-03 UNITS INCH.
LAVIS, PARAM 4, 4TH ØRDER T/A F.B., MAG CURVES
ØRDINATE=ALPHA, ABCISSA=ØMEGA, CØNBETA=0.046

FIGURE 5-7



X-SCALE=1.00E+01 UNITS INCH.
Y-SCALE=5.00E-03 UNITS INCH.
GLAVIS, PARAM 4, 4TH ORDER T/A F.B., MAG CURVES
ORDINATE=BETA , ABCISSA=OMEGA, CONALPHA=0.00087

FIGURE 5-8



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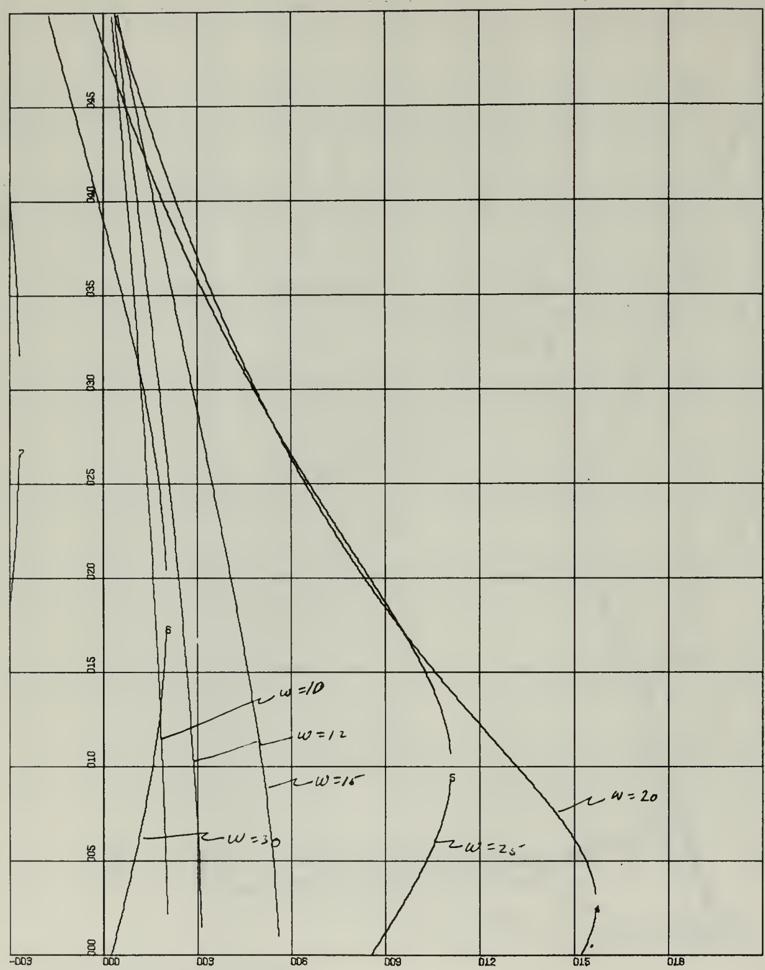
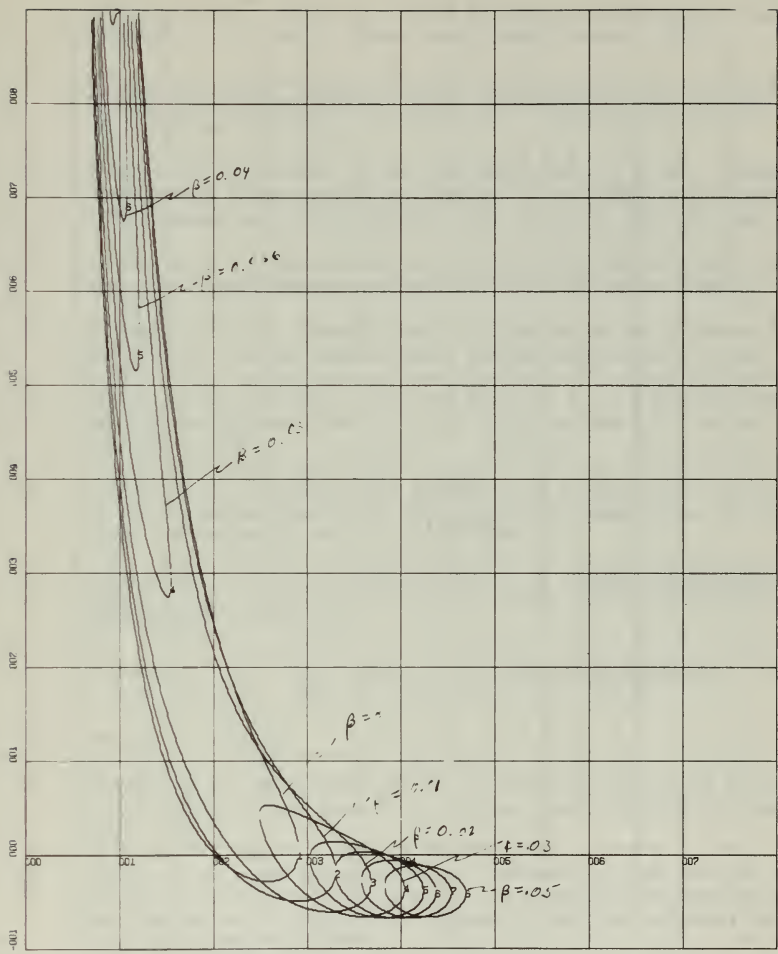
Y-SCALE=1.00E-03 UNITS INCH.
CLAUIS, PARAM 5, 4TH ØRDER T/A F.B.,ØMEGA CURVES
ØRDINATE=ALPHA, ABCISSA=MAG , CØNBETA=0.046

FIGURE 5-9



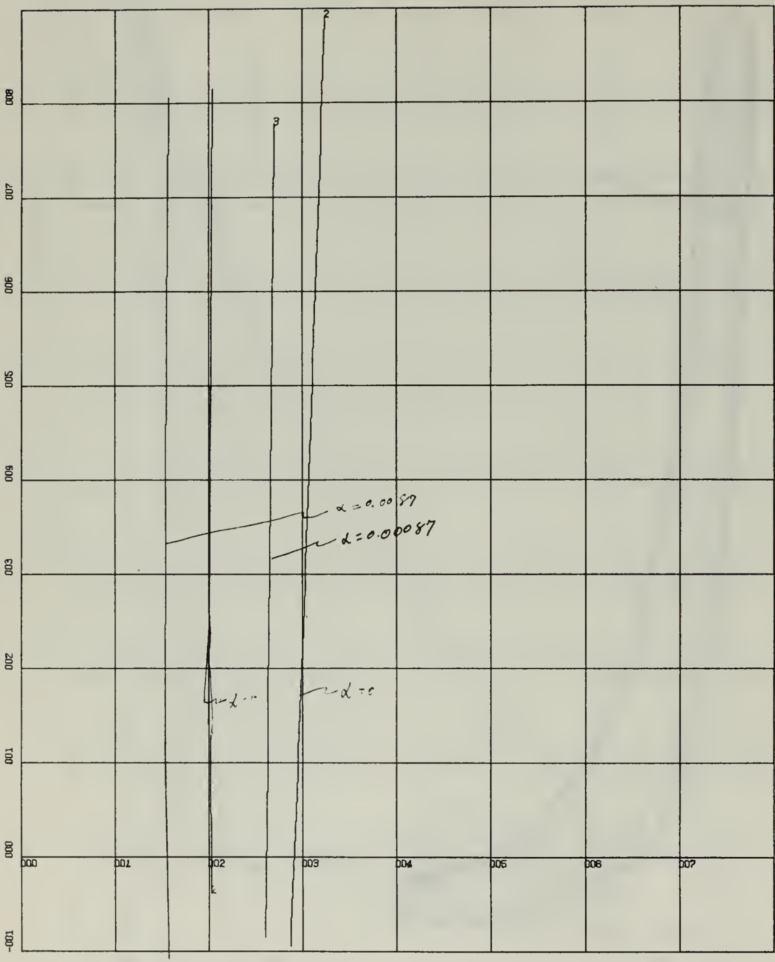
X-SCALE=3.00E+00 UNITS INCH.
Y-SCALE=5.00E-03 UNITS INCH.
GLAVIS, PARAM 5, 4TH ORDER T/A F.B.,OMEGA CURVES
ORDINATE=BETA , ABCISSA=MAG , CONALPHA=0.00087

FIGURE 5-10



X-SCALE=1.00E+01 UNITS INCH.
Y-SCALE=1.00E-03 UNITS INCH.
GLAVIS, PARAM 6, 4TH ØRDER T/A F.B., BETA CURVES
ØRDINATE=~~BETA~~ ALPHA, ABCISSA=ØMEGA, CØNMAGNITUDE=6DB

FIGURE 5-11



X-SCALE=1.00E+01 UNITS INCH.
Y-SCALE=1.00E-03 UNITS INCH.
GLAVIS, PARAM 6, 4TH ORDER T/A F.B.,ALPHA CURVES
ORDINATE=ALPHA, ABCISSA=OMEGA, CONMAGNITUDE=6DB

FIGURE 5-12

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12. Thaler, G. J., and Thompson, A. G., "Parameter Plane Methods for the Study of the Frequency Response of Linear Time-Invariant Systems", The Institution of Engineers, Australia, Applied Mechanics Conference Paper No. 2292, June 1967.

APPENDIX I

```
C******* PROGRAM PARAMS **********
C  THIS PROGRAM CONVERTS A GIVEN TRANSFER FUNCTION TO A SET OF FREQUENCY
CDOMAIN REPRESENTATIONS FOR ANALYSIS AND DESIGN STUDIES. THIS TRANSFER
CFUNCTION IS READ AS A RATIO OF TWO COMPLEX FREQUENCY POLYNOMIALS, WITH
CTWO VARIABLE PARAMETERS ALPHA AND BETA EMBEDDED IN THE COEFFICIENTS.
CTHE PROGRAM THEN TRANSFORMS THIS  TRANSFER FUNCTION INTO THE ANGULAR
CFREQUENCY DOMAIN, YIELDING AN EQUATION IN TERMS OF FOUR VARIABLES:
CALPHA, BETA, MAGNITUDE, AND OMEGA. SEVEN TWO-DIMENSIONAL GRAPHICAL
CREPRESENTATIONS ARE GENERATED BY FIXING ONE VARIABLE AS A CONSTANT;
CASSIGNING DESIRED CURVE VALUES TO ANOTHER; AND LETTING THE REMAINING
CTWO VARIABLES BE THE ORDINATE AND ABCISSA VALUES. A THIRD COEFFICIENT
CVARIABLE PARAMETER, GAMMA, MAY ALSO BE EMBEDDED INTO THE POLYNOMIAL.
CIN THIS CASE, THE ABOVE PROCESS IS REPEATED FOR EACH VALUE OF THE THIRD
CVARIABLE.              .....  DATA CARD INFORMATION FOLLOWS  ......
C*******************DATA CARD INFORMATION  SECTION*************************
CCARD FORMAT   SYMBOL        USAGE OF SYMBOL
C  1    8I10   NRUN      NUMBER OF DESIRED RUNS. EACH MUST HAVE A COMPLETE
C                        SET OF DATA CARDS LISTED BELOW. NOT EVERY SYMBOL
C                        IS USED IN ALL CASES. LEAVE UNUSED SYMBOLS BLANK.
C                        DO NOT OMIT ANY BLANK CARDS, UNLESS SO SPECIFIED.
C                        INSERT ADDITIONAL CARDS BEHIND EACH FULL CARD.
C  2    8I10   KPARAM    NUMBER OF GRAPHICAL PRESENTATIONS TO BE PROGRAMMED
C              JPARAM    NUMERICAL TITLES (1 TO 7) OF EACH PARAM TO BE RUN;
C                        MUST AGREE IN NUMBER WITH KPARAM ABOVE.
C  3    8I10   NORN      ORDER OF NUMERATOR POLYNOMIAL (LIMITED TO 49)
C              NORD      ORDER OF DENOMINATOR POLYNOMIAL (LIMITED TO 49)
C              MODORD    Y-AXIS VARIABLE, AND CURVE VARIABLE CONTROL
C                        MODORD=1: ALPHA IS ORDINATE VARIABLE (PARAM 1-6)
C                        MODORD=2: BETA  IS ORDINATE VARIABLE (PARAM 1-6)
C                        MODORD=1: ALPHA CURVES WILL BE PLOTTED (PARAM 7)
C                        MODORD=2: BETA CURVES WILL BE PLOTTED (PARAM 7)
C              NALPHA    NUMBER OF ALPHA CURVES DESIRED (LIMITED TO 16)
C              NBETA     NUMBER OF BETA CURVES DESIRED (LIMITED TO 16)
C              NMAG      NUMBER OF MAGNITUDE CURVES DESIRED (LIMITED TO 16)
C              NOMEGA    NUMBER OF OMEGA CURVES DESIRED (LIMITED TO 16)
C              NGAMMA    POSITIVE INTEGER DENOTES NUMBER OF THIRD PARAMETER
C                        VALUES TO BE EMBEDDED INTO EACH PARAM RUN. INSERT
C                        COEFFICIENTS FOR THESE GAMMA VALUES ON SEPARATE
C                        CARDS DIRECTLY BEHIND REST OF COEFFICIENTS; SEE#19
C                        NEGATIVE INTEGER DENOTES NUMBER OF PARAM-7 VECTOR
C                        ADDITIONS FOR GENERALIZED BLOCK DIAGRAM STUDIES.
C                        THE (NEGATIVE) VALUE OF NGAMMA DETERMINES HOW MANY
C                        BODE CALCULATIONS WILL BE MADE BEFORE PLOTTING THE
C                        VECTOR RESULTANT AS ONE MAGNITUDE, AND ONE PHASE.
C                        INSERT ADDITIONAL SETS OF COEFFICIENTS AT END .
C  4    8I10   IMAG      MAGNITUDE VALUE FORM CONTROL
C                        IMAG=0: ALL VALUES IN DECIBELS (PARAM 7 THIS FORM)
C                        IMAG=1 MEANS ALL VALUES IN ACTUAL MAGNITUDE
C                        IMAG=2 MEANS ALL VALUES IN MAGNITUDE SQUARED
C              IWRITE    PRINTOUT CONTROL
C                        IWRITE=-1 MEANS NO PRINTOUT
C                        IWRITE=0 MEANS ONLY INPUT DATA WILL BE PRINTED
C                        IWRITE=1 TO 900 YIELDS INPUT DATA, GRAPHICAL
C                        INFORMATION, AND COMPUTATIONAL RESULTS. THE NUMBER
C                        OF COMPUTATIONS BETWEEN EACH PRINTOUT IS DICTATED
C                        BY THE VALUE  OF IWRITE. IF IWRITE IS GREATER THAN
C                        900, ONLY INPUT DATA & GRAPH INFO. WILL BE PLOTTED
C              IGRAPH    FREQUENCY SCALING CONTROL
C                        IGRAPH=-2: NO GRAPHS WILL BE PLOTTED.
C                        IGRAPH=-1:ONLY PARAM 7 WILL BE PLOTTED ON LOG GRID
C                        IGRAPH=0: PARAMS 6&7 WILL BE PLOTTED ON LOG GRID.
C                        IGRAPH=1:PARAMS 4,6,&7 WILL BE PLOTTED ON LOG GRID
C                        IGRAPH=2: PARAMS 4&7 WILL BE PLOTTED ON LOG GRID.
C              IPHASE    PARAM 7 PHASE PLOT CONTROL
C                        IPHASE=-1 MEANS NO PHASE CURVES
C                        IPHASE=0: PHASE CURVES SUPERIMPOSED ON MAGNITUDE
C                        IPHASE=1: PHASE CURVES PLOTTED ON SEPARATE GRAPH
C              IQUAD     PARAM 7 PHASE PLOT SCALING CONTROL
C                        IQUAD=0 YIELDS PHASE CURVES FROM 0 TO -180 DEGREES
C                        IQUAD=1 YIELDS PHASE CURVES FROM-90 TO +90 DEGREES
C              IPOLAR    PARAM 7 POLAR PLOT CONTROL
C                        IPOLAR=0 MEANS NO POLAR PLOT
C                        IPOLAR=1 GIVES POLAR PLOT ON SEPARATE GRAPH
C              LX        LENGTH OF X-AXIS  (LIMITED TO 9 INCHES)
C              LY        LENGTH OF Y-AXIS  (LIMITED TO 15 INCHES)
C  5   8E10.3  CONALP    VALUE OF ALPHA WHEN USED AS A CONSTANT
C              CONBET    VALUE OF BETA WHEN USED AS A CONSTANT
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C                 CONMAG   MAGNITUDE VALUE WHEN USED AS A CONSTANT: SEE IMAG
C                 CONOME   VALUE OF OMEGA WHEN USED AS A CONSTANT (NOT ZERO)
C****************DATA CARD INFORMATION SECTION (CONTINUED)****************
CCARD FORMAT    SYMBOL         USAGE OF SYMBOL
C   6  8E10.3   ALPHMN    MIN. VALUE OF ALPHA WHEN USED AS AN AXIS VARIABLE
C               ALPHMX    MAX. VALUE OF ALPHA WHEN USED AS AN AXIS VARIABLE
C               BETAMN    MIN. VALUE OF BETA WHEN USED AS AN AXIS VARIABLE
C               BETAMX    MAX. VALUE OF BETA WHEN USED AS AN AXIS VARIABLE
C               XMAGMN    MIN. MAG. VALUE WHEN USED AS AXIS VAR.: SEE IMAG
C               XMAGMX    MAX. MAG. VALUE WHEN USED AS AXIS VAR.: SEE IMAG
C               OMEGMN    MIN. OMEGA VALUE WHEN USED AS AXIS VARIABLE
C               OMEGMX    MAX. OMEGA VALUE WHEN USED AS AXIS VARIABLE
C                         NOTE:IF LOG GRID IS REQUESTED,(SEE IGRAPH), OMEGMN
C                         WILL BE AUTO. CHANGED TO NEAREST POWER OF 10, AND
C                         OMEGMX CHANGED LIKEWISE. THE NUMBER OF DECADES IS
C                         AUTOMATICALLY COMPUTED FROM THESE NEW VALUES.
C   7  8E10.3   ZALPHA    DESIRED VALUES FOR ALPHA CURVES (SEE NALPHA)
C   8  8E10.3   ZBETA     DESIRED VALUES FOR BETA CURVES (SEE NBETA)
C   9  8E10.3   ZMAG      DESIRED VALUES FOR MAGNITUDE CURVES(SEE NMAG&IMAG)
C  10  8E10.3   ZOMEGA    DESIRED POSITIVE OMEGA CURVE VALUES (SEE NOMEGA)
C  11  8E10.3   HA        NUMERATOR CONSTANT COEFFICIENTS IN ASCENDING ORDER
C  12  8E10.3   HAT       NUMERATOR ALPHA COEFFICIENTS IN ASCENDING ORDER
C  13  8E10.3   HBT       NUMERATOR BETA COEFFICIENTS IN ASCENDING ORDER
C  14  8E10.3   HCT       NUM. ALPHA-BETA COEFFICIENTS IN ASCENDING ORDER
C  15  8E10.3   HD        DENOMINATOR CONST. COEFFICIENTS IN ASCENDING ORDER
C  16  8E10.3   HDT       DENOMINATOR ALPHA  COEFFICIENTS IN ASCENDING ORDER
C  17  8E10.3   HET       DENOMINATOR BETA COEFFICIENTS IN ASCENDING ORDER
C  18  8E10.3   HFT       DENOM. ALPHA-BETA COEFFICIENTS IN ASCENDING ORDER
C * * * * * * * * * *    OMIT CARDS # 19 THRU # 27 IF NGAMMA LESS THAN ONE.
C  19  8E10.3   ZGAMMA    VALUES OF THE THIRD PARAMETER; MUST MATCH NGAMMA
C  20  8E10.3   GAMHA     NUM. GAMMA-CONST.  COEFFICIENTS IN ASCENDING ORDER
C  21  8E10.3   GAMHAT    NUM. GAMMA-ALPHA   COEFFICIENTS IN ASCENDING ORDER
C  22  8E10.3   GAMHBT    NUM. GAMMA-BETA    COEFFICIENTS IN ASCENDING ORDER
C  23  8E10.3   GAMHCT    NUM. GAMMA-ALPHA-BETA COEFFS.   IN ASCENDING ORDER
C  24  8E10.3   GAMHD     DEN. GAMMA-CONST. COEFFICIENTS IN ASCENDING ORDER
C  25  8E10.3   GAMHDT    DEN. GAMMA-ALPHA  COEFFICIENTS IN ASCENDING ORDER
C  26  8E10.3   GAMHET    DEN. GAMMA-BETA   COEFFICIENTS IN ASCENDING ORDER
C  27  8E10.3   GAMHFT    DEN. GAMMA-ALPHA-BETA COEFFS.  IN ASCENDING ORDER
C  28  6A8       ITITLE   FIRST LINE GRAPH TITLE (COLUMNS 1THRU48)
C  29  6A8       ITITLE   SECOND LINE OF GRAPH TITLE (COLUMNS 1THRU48)
C                         THE NUMBER OF TITLES FOR PARAM 7 MUST MATCH THE
C                         NUMBER OF GRAPHS REQUESTED.(SEE IPHASE & IPOLAR)
C  30  6A8       JTITLE   1ST 1/2 PHASE TITLE:OMIT UNLESS NPARAM=7&IPHASE=1
C  31  6A8       JTITLE   2ND 1/2 PHASE TITLE:OMIT UNLESS NPARAM=7&IPHASE=1
C  32  6A8       KTITLE   1ST 1/2 POLAR TITLE:OMIT UNLESS NPARAM=7&IPOLAR=1
C  33  6A8       KTITLE   2ND 1/2 POLAR TITLE:OMIT UNLESS NPARAM=7&IPOLAR=1
C***********************************************************************
C***********************************************************************
      REAL*8ITITLE(12),JTITLE(12),KTITLE(12),JOKER(9)/1H ,6HPARAM-,7HFUR
     &THER,5HALPHA,4HBETA,5HOMEGA,8HMAGNITUD,5HGAMMA,8HSEPARATE/
      REAL DM/1H1/,LA(66)/4H 0.1,4H 0.2,4H 0.3,4H 0.4,4H 0.5,4H 0.6,4H 0
     &.7,4H 0.8,4H 0.9,4H 1.0,4H 1.0,4H 2.0,4H 3.0,4H 4.0,4H 5.0,4H 6.0,
     &4H 7.0,4H 8.0,4H 9.0,4H10.0,4H10.0,4H20.0,4H30.0,4H40.0,4H50.0,
     & 4H60.0,4H70.0,4H80.0,4H90.0,4H100.,4H100.,4H200.,4H300.,4H400.,
     &4H500.,4H600.,4H700.,4H800.,4H900.,4H1000,4H1000,4H2000,4H3000,
     &4H4000,4H5000,4H6000,4H7000,4H8000,4H9000,1H ,1H1,1H2,1H3,1H4,1H5,
     &1H6,1H7,1H8,1H9,2H10,2H11,2H12,2H13,2H14,2H15,2H16/
      DIMENSION JPARAM(8),A(50),B(50),D(50),F(50),HA(50),HAT(50),HBT(50)
     &,HCT(50),HD(50),HDT(50),HET(50),HFT(50),OLDHA(50),OLDHAT(50),OLDHB
     &T(50),OLDHCT(50),OLDHD(50),OLDHDT(50),OLDHET(50),OLDHFT(50),GAMHA(
     &50),GAMHAT(50),GAMHBT(50),GAMHCT(50),GAMHD(50),GAMHDT(50),GAMHET(5
     &0),GAMHFT(50),IC(16),ZALPHA(16),ZBETA(16),ZGAMMA(16),ZMAGSQ(16),ZO
     &MEGA(16),S(16,900),T(16,900),U(16,900),V(16,900),W(16,900),X(900),
     &Y(900)
      CALL CANCEL(2)
      EQUIVALENCE(S,T,U,V,W)
    1 FORMAT(8I10)
    2 FORMAT(10X,8I10)
    3 FORMAT(6A8)
    4 FORMAT(10X,6A8)
    5 FORMAT(   8E10.3)
    6 FORMAT(10X,1P10E12.3)
C**************************INPUT DATA READ IN **************************
      READ(5,1)NRUN
      DO 1000 IRUN=1,NRUN
      READ(5,1) KPARAM,(JPARAM(I),I=1,KPARAM)
      READ(5,1)NORN,NORD,MODORD,NALPHA,NBETA,NMAG,NOMEGA,NGAMMA
      READ(5,1) IMAG ,IWRITE ,IGRAPH,IPHASE,IQUAD,IPOLAR,LX,LY
      READ(5,5)CONALP,CONBET,CONMAG,CONOME
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      READ(5,5)ALPHMN,ALPHMX,BETAMN,BETAMX,XMAGMN,XMAGMX,OMEGMN,OMEGMX
      READ(5,5) (ZALPHA(I),I=1,NALPHA)
      READ(5,5) (ZBETA(I),I=1,NBETA)
      READ(5,5) (ZGAMMA(I),I=1,NGAMMA)
      READ(5,5) (ZMAGSQ(I),I=1,NMAG)
      READ(5,5) (ZOMEGA(I),I=1,NOMEGA)
      LOE=NORN+1
      MOE=NORD+1
      READ(5,5) (HA (I), I=1,LOE)
      READ(5,5) (HAT(I), I=1,LOE)
      READ(5,5) (HBT(I), I=1,LOE)
      READ(5,5) (HCT(I), I=1,LOE)
      READ(5,5) (HD (I), I=1,MOE)
      READ(5,5) (HDT(I), I=1,MOE)
      READ(5,5) (HET(I), I=1,MOE)
      READ(5,5) (HFT(I), I=1,MOE)
      IF((NGAMMA).GE.(1)) READ(5,5) (ZGAMMA(I),I=1,NGAMMA)
      IF((NGAMMA).GE.(1)) READ(5,5) (GAMHA (I),I=1,LOE)
      IF((NGAMMA).GE.(1)) READ(5,5) (GAMHAT(I),I=1,LOE)
      IF((NGAMMA).GE.(1)) READ(5,5) (GAMHBT(I),I=1,LOE)
      IF((NGAMMA).GE.(1)) READ(5,5) (GAMHCT(I),I=1,LOE)
      IF((NGAMMA).GE.(1)) READ(5,5) (GAMHD (I),I=1,MOE)
      IF((NGAMMA).GE.(1)) READ(5,5) (GAMHDT(I),I=1,MOE)
      IF((NGAMMA).GE.(1)) READ(5,5) (GAMHET(I),I=1,MOE)
      IF((NGAMMA).GE.(1)) READ(5,5) (GAMHFT(I),I=1,MOE)
      IF((IWRITE).LT.(0)) GO TO 24
C*************************INPUT DATA PRINT OUT SECTION*****************
      WRITE(6,7) IRUN,NORN,NORD,KPARAM,(JOKER(2),JPARAM(I),I=1,KPARAM)
    7 FORMAT(1H1,10X,'*****  THE INPUT DATA FOR RUN',I2,' IS  *****',//,
     &10X,'ORDER OF NUMERATOR:',I2,//,10X,'ORDER OF DENOMINATOR:',I2,//,
     &10X,'THE FOLLOWING',I2,' GRAPHICAL CONFIGURATIONS WILL BE COMPUTED
     &:',//,20X,8(A8,I1,3X),/)
      IF((NGAMMA).GE.(1)) WRITE(6,8) NGAMMA,(ZGAMMA(I),I=1,NGAMMA)
    8 FORMAT(10X,I2,' GAMMA, OR THIRD VAR. PARAMETER COEFFICIENTS WILL B
     &E USED.',//,10X,'THE GAMMA VALUES ARE:',1P8E12.3,/,10X,1P8E12.3,/)
      WRITE(6,10)
   10 FORMAT(10X,'THE COEFFICIENTS FOR THE NUMERATOR ARE:',/)
      DO 11 I=1,LOE
      J=I-1
      IF(NGAMMA .LE.0) WRITE(6,12) HA(I),HAT(I),HBT(I),HCT(I),J
   12 FORMAT(20X,'(',1PE10.3,'+',1PE10.3,'*ALPHA + ' ,1PE10.3,'*BETA + '
     &,1PE10.3,'*ALPHA*BETA ) * S**(',I2,')',/)
      IF(NGAMMA .GT.0) WRITE(6, 9) HA(I),GAMHA(I),HAT(I),GAMHAT(I),HBT(I
     &),GAMHBT(I),HCT(I),GAMHCT(I),J
    9 FORMAT(15X,'((',1PE10.3,'+',1PE10.3,'*GAMMA)+(',1PE10.3,'+',1PE10.
     &3,'*GAMMA)*ALPHA ....',//,20X,'+(',1PE10.3,'+',1PE10.3,'*GAMMA)*BE
     &TA+(',1PE10.3,'+',1PE10.3,'*GAMMA)*ALPHA*BETA)**(',I2,')',/)
   11 CONTINUE
      WRITE(6,13)
   13 FORMAT(/10X,'THE COEFFICIENTS FOR THE DENOMINATOR ARE:',/)
      DO 14 I=1,MOE
      J=I-1
      IF(NGAMMA .LE.0) WRITE(6,12) HD(I),HDT(I),HET(I),HFT(I),J
      IF(NGAMMA .GT.0) WRITE(6, 9) HD(I),GAMHD(I),HDT(I),GAMHDT(I),HET(I
     &),GAMHET(I),HFT(I),GAMHFT(I),J
   14 CONTINUE
      WRITE(6,15)
   15 FORMAT(15X,'  NORN        NORD      MODORD      NALPHA      NBETA      NMA
     &G        NOMEGA   NGAMMA ',/)
      WRITE(6,2 ) NORN,NORD,MODORD,NALPHA,NBETA,NMAG,NOMEGA,NGAMMA
      WRITE(6,16)
   16 FORMAT(/,17X,'IMAG        IWRITE      IGRAPH      IPHASE      IQUAD      IPC
     &LAR     LXAXIS     LYAXIS  ',/)
      WRITE(6,2)  IMAG ,IWRITE,IGRAPH,IPHASE,IQUAD,IPOLAR,LX,LY
      WRITE(6,17)
   17 FORMAT(/,12X,' CCNALPHA      CONBETA      CONMAG      CONOMEGA  ',/)
      WRITE(6,6 )CONALP,CONBET,CONMAG,CONOME
      WRITE(6,18)
   18 FORMAT(/,12X,' ALPHMIN      ALPHMAX      BETAMIN      BETAMAX      XMAG
     &MIN     XMAGMAX       OMEGAMIN       OMEGAMAX   ',/)
      WRITE(6,6)ALPHMN,ALPHMX,BETAMN,BETAMX,XMAGMN,XMAGMX,OMEGMN,OMEGMX
      WRITE(6,19) JOKER(4),(ZALPHA(I),I=1,NALPHA)
      WRITE(6,19) JOKER(5),(ZBETA (I),I=1,NBETA )
      WRITE(6,19) JOKER(7),(ZMAGSQ(I),I=1,NMAG)
      WRITE(6,19) JOKER(6),(ZOMEGA(I),I=1,NOMEGA)
   19 FORMAT(/,10X,A8,'CURVE VALUES:',1P8E12.3,/31X,1P8E12.3)
      IF(IWRITE) 20,20,22
   20 WRITE(6,21)
   21 FORMAT(/,10X,'THERE WILL BE NO PRINTOUT OF COMPUTED RESULTS')
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      GO TO 24
   22 WRITE(6,23)IWRITE
   23 FORMAT(/,10X,'COMPUTED RESULTS WILL BE PRINTED EVERY',I3,' STEPS')
C*******************INITIALIZING COMPUTATIONAL SETTINGS*******************
   24 IF(MODORD.GE.2) GO TO  25
      DUMMY1=BETAMN
      DUMMY2=BETAMX
      BETAMN=ALPHMN
      BETAMX=ALPHMX
      ALPHMN=DUMMY1
      ALPHMX=DUMMY2
   25 DO 1000 IPARAM=1,KPARAM
      NPARAM=JPARAM(IPARAM)
      MGAMMA=NGAMMA+1
      DO 1000 IGAMMA=1,MGAMMA
      GAMMA=ZGAMMA(IGAMMA)
      IF(NGAMMA.GE.1) GO TO 517
   26 XS=(ALPHMX-ALPHMN)/LX
      IF((ABS(XS)).LE.(1.0E-10)) XS=1.0E-10
      YS=(BETAMX-BETAMN)/LY
      IF((ABS(YS)).LE.(1.0E-10)) YS=1.0E-10
      IX=-BETAMN/YS
      IY=-ALPHMN/XS
      MOD=1
      JCURVE=0
      IPAR7=1
      READ(5,3)(ITITLE(I),I=1,12)
      IF(NPARAM.NE.7)GO TO 34
      IF(IMAG-1)27,28,29
   27 DBMAX=XMAGMX
      DBMIN=XMAGMN
      GO TO 30
   28 DBMAX=20*ALOG10(XMAGMX)
      DBMIN=20*ALOG10(XMAGMN)
      GO TO 30
   29 DBMAX=10*ALOG10(XMAGMX)
      DBMIN=10*ALOG10(XMAGMN)
   30 XS=(DBMAX-DBMIN)/LX
      IY=DBMAX/XS
      DO 31 I=1,16
      S(I,1)=0.0
      T(I,1)=0.0
      U(I,1)=0.0
      V(I,1)=0.0
   31 W(I,1)=0.0
      IF(IPHASE.EQ.0) GO TO 32
      READ(5,3)(JTITLE(I),I=1,12)
   32 IF((IPOLAR).LE.(0)) GO TO 33
      READ(5,3)(KTITLE(I),I=1,12)
   33 IF(MODORD-2)45,44,44
   34 GO TO (35,40,41,46,47,51,59,59),NPARAM
   35 NLIMIT=NOMEGA
   36 IF(IMAG-1)37,38,39
   37 DUMMY1=10.0**(CONMAG/10.0)
      IF(NPARAM-1)59,59,43
   38 DUMMY1=CONMAG*CONMAG
      IF(NPARAM-1)59,59,43
   39 DUMMY1=CONMAG
      IF(NPARAM-1)59,59,43
   40 NLIMIT=NMAG
   41 OMEGA=CONOME
      IF(NPARAM-2)59,59,42
   42 XS=(XMAGMX-XMAGMN)/LX
      IY=-XMAGMN/XS
   43 IF(MODORD-2)44,45,45
   44 NLIMIT=NBETA
      IF(NPARAM-6)59,53,55
   45 NLIMIT=NALPHA
      IF(NPARAM-6)59,53,55
   46 NLIMIT=NMAG
      XS=(OMEGMX-OMEGMN)/LX
      IY=-OMEGMN/XS
   47 IF(MODORD-2)48,49,49
   48 CONST=CONBET
      IF(NPARAM-4)52,52,50
   49 CONST=CONALP
      IF(NPARAM-4)52,52,50
   50 XS=(XMAGMX-XMAGMN)/LX
      IY=-XMAGMN/XS
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      NLIMIT=NOMEGA
      GO TO 59
   51 XS=(OMEGMX-OMEGMN)/LX
      IY=-OMEGMN/XS
      GO TO 36
   52 IF(IGRAPH)59,59,54
   53 IF((IGRAPH.LT.0).OR.(IGRAPH.GT.1)) GO TO 59
   54 IY=LX-IX
   55 IF(IGRAPH.LE.-2) GO TO 59
C********************LOG GRID GRAPHICAL CONSTRUCTION*********************
      IF(OMEGMN.LT.1000.)YLOGMN=100.
      IF(OMEGMN.LT.100. )YLOGMN=10.
      IF(OMEGMN.LT.10.  )YLOGMN=1.
      IF(OMEGMN.LT.1.   )YLOGMN=0.1
      IF(OMEGMX.GT.1.0  )YLOGMX=10.0
      IF(OMEGMX.GT.10.0)YLOGMX=100.0
      IF(OMEGMX.GT.100.)YLOGMX=1000.0
      IF(OMEGMX.GT.1000.)YLOGMX=10000.0
      DUMMY3= YLOGMX/YLOGMN
      NUMDEC=ALOG10(DUMMY3)
      IX=0
      DUMMY4=NUMDEC
      DUMMY5=LY
      YS=DUMMY4/DUMMY5
      IFINAL=LX+1
      JFINAL=LY+1
      DO 57  IA=1,IFINAL
      DO 56  JA=1,JFINAL
      X(JA)=XS*(IA-1-IY)
   56 Y(JA)=YS*(JA-1)
      CALL DRAW(JA,X,Y,MOD,0,LA(50),ITITLE,XS,YS ,0,IY,2,2,LX,LY,0,LAST)
   57 MOD=2
      DO 58 IB=1,NUMDEC
      DO 58 JB=1,10
      J=ALOG10(YLOGMN)*10.+10.+(IB-1)*10.+JB
      YLOG=JB
      DO 1058 KB=1,IFINAL
      X(KB)=XS*(KB-1-IY)
 1058 Y(KB)=ALOG10(YLOG)+IB-1
   58 CALL DRAW(KB,X,Y,MOD,0,LA(J),ITITLE,XS,YS ,0,IY,2,2,LX,LY,0,LAST)
      IF((IPAR7).NE.(1)) GO TO 530
   59 IF(IWRITE.LE.0) GO TO 200
C******************GRAPHICAL  INFORMATION PRINTOUT SECTION**************
      WRITE(6,60) IRUN, NPARAM
   60 FORMAT(1H1,10X,'**********THE PRELIMINARY DATA FOR RUN NUMBER',I2,
     &', PARAMETER NUMBER',I2,' IS    **********',///)
      IF(NGAMMA.GE.1) WRITE (6,61) ZGAMMA(IGAMMA)
   61 FORMAT(10X,'THE FOLLOWING CALCULATIONS WILL BE BASED ON THE THIRD
     &VARIABLE PARAMETER VALUE , GAMMA=',1PE12.5,//)
      WRITE (6,62) (ITITLE(I),I=1,12)
   62 FORMAT(10X,'THE GRAPH WILL BE TITLED:',//,10X,6A8,//,10X,6A8,/)
      IF(NPARAM.NE.7) GO TO 80
      WRITE(6,83)
      WRITE(6,   63)DBMIN,DBMAX,XS
   63 FORMAT(10X,'MAGNITUDE, THE DEPENDENT VARIABLE, WILL BE PLOTTED ON
     &THE ORDINATE FROM',F10.4,' TO',F10.4,' AT',F10.4,' DB/DIVISION',/)
      IF(IPHASE) 73,64,66
   64 WRITE(6,65)
   65 FORMAT(10X,'PHASE CURVES WILL BE PLOTTED ON MAGNITUDE GRAPH',/)
      GO TO 68
   66 WRITE(6,67)
   67 FORMAT(10X,'PHASE CURVES WILL BE PLOTTED ON SEPARATE GRAPH',/)
   68 IF(IQUAD.GT.0) GO TO 70
      WRITE(6,69)
   69 FORMAT(10X,'FROM ZERO DEGREES THROUGH -90 TO -180 DEGREES',/)
      GO TO 72
   70 WRITE(6,71)
   71 FORMAT(10X,'FROM -90 DEGREES THROUGH -180 TO -270 DEGREES',/)
   72 WRITE(6,62) (JTITLE(I),I=1,12)
   73 IF(IPOLAR.LE.0) GO TO 75
      WRITE(6,74) XS
   74 FORMAT(10X,'A POLAR PLOT OF MAGNITUDE VS. PHASE WILL BE PLOTTED, W
     &ITH MAGNITUDE SCALED AT',F10.5,' DECIBELS  PER HALF INCH',/)
      WRITE(6,62) (KTITLE(I),I=1,12)
   75 IF(MODORD-2)127,121,121
   76 FORMAT(10X,A8,', THE   DEPENDENT VARIABLE, WILL BE PLOTTED ON THE
     &ORDINATE FROM',1PE10.3,' TO',1PE10.3,' WITH A SCALE OF',1PE10.3,/)
   77 FORMAT(10X,A8,', THE INDEPENDENT VARIABLE, WILL BE PLOTTED ON THE
     &ABCISSA, FROM',1PE10.3,' TO',1PE10.3,' WITH A SCALE OF',1PE10.3,/)
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 78 FORMAT(10X,A8,' WILL BE HELD CONSTANT AT THE VALUE OF',1PE12.5,/)
 79 FORMAT(10X,A8,' WILL BE GRAPHED AS ',I3,' CURVES, CONSISTING OF TH
   &E FOLLOWING VALUES:',16('CURVE',I3,' = ',1PE12.5,/))
 80 IF(IMAG.EQ.0) WRITE(6,83)
    IF(IMAG.EQ.1) WRITE(6,84)
    IF(IMAG.EQ.2) WRITE(6,85)
 81 IF(MODORD-2) 82,  94,1000
 82 WRITE(6,76) JOKER(4),BETAMN,BETAMX, YS
 83 FORMAT(10X,'ALL MAGNITUDE COMPUTATIONS WILL BE IN DECIBELS',/)
 84 FORMAT(10X,'ALL MAGNITUDE COMPUTATIONS ARE ACTUAL MAGNITUDE',/)
 85 FORMAT(10X,'ALL MAGNITUDE COMPUTATIONS ARE MAGNITUDE SQUARED',/)
    GO TO (86,86,121,130,130,121,200,200),NPARAM
 86 WRITE(6,77) JOKER(5),ALPHMN,ALPHMX,XS
    IF(NPARAM-1) 88,88,99
 88 WRITE(6,78) JOKER(7),CONMAG
 90 WRITE(6,79)JOKER(6),NOMEGA,(I,ZOMEGA(I),I=1,NOMEGA)
    IF(NPARAM-5)200,125,200
 94 WRITE(6,76) JOKER(5),BETAMN,BETAMX, YS
 96 GO TO(97,97,127,111,111,127,200,200),NPARAM
 97 WRITE(6,77) JOKER(4),ALPHMN,ALPHMX,XS
    IF(NPARAM-1)88,88,99
 99 WRITE(6,78) JOKER(6),CONOME
101 WRITE(6,79)JOKER(7),NMAG,(I,ZMAGSQ(I),I=1,NMAG)
    IF(NPARAM-4)200,139,200
111 WRITE(6,78) JOKER(4),CONALP
113 WRITE(6,114)
114 FORMAT(10X,'THE COMBINED COEFFICIENTS OF THE NUMERATOR ARE',/)
115 DO 116 I=1,LOE
    A(I)=HA(I)+HAT(I)*CONST
    B(I)=HBT(I)+HCT(I)*CONST
    KA=I-1
116 WRITE(6,117) A(I),B(I),KA
117 FORMAT(20X,1H(,E10.4,3H + , E10.4,16H * BETA ) * S**(,I2,1H)/)
    WRITE(6,136)
    DO 119 I=1,MOE
    D(I)=HD(I)+HDT(I)*CONST
    E(I)=HET(I)+HFT(I)*CONST
    KA=I-1
119 WRITE(6,117) D(I),E(I),KA
120 GO TO (200,200,200,101, 90,200,139,200),NPARAM
121 WRITE(6,79)JOKER(5),NBETA,(I,ZBETA(I),I=1,NBETA)
    IF(NPARAM-6)124,138,111
124 WRITE(6,78) JOKER(7),CONOME
125 WRITE(6,77) JOKER(7),XMAGMN,XMAGMX,XS
    GO TO 200
127 WRITE(6,79)JOKER(4),NALPHA,(I,ZALPHA(I),I=1,NALPHA)
    IF(NPARAM-6)124,138,130
130 WRITE(6,78) JOKER(5),CONBET
131 FORMAT(10X,'BETA  WILL BE HELD CONSTANT AT THE  VALUE OF',F10.4,/)
132 WRITE(6,114)
    DO 134 I=1,LOE
    A(I)=HA(I)+HBT(I)*CONST
    B(I)=HAT(I)+HCT(I)*CONST
    KA=I-1
134 WRITE(6,135)A(I),B(I),KA
135 FORMAT(20X,1H(,E10.4,3H + ,E10.4,17H * ALPHA ) * S**(,I2,1H)/)
    WRITE(6,136)
136 FORMAT(10X,'THE COMBINED COEFFICIENTS OF THE DENOMINATOR ARE-',/)
    DO 137 I=1,MOE
    D(I)=HD(I)+HET(I)*CONST
    E(I)=HDT(I)+HFT(I)*CONST
    KA=I-1
137 WRITE(6,135) D(I),E(I),KA
    GO TO 120
138 WRITE(6,78) JOKER(7),CONMAG
139 WRITE(6,140)
140 FORMAT(10X,'OMEGA,THE INDEPENDENT VARIABLE,WILL BE PLOTTED ON THE
   &ABCISSA',/)
    IF(NPARAM-6)141,143,144
141 IF(IGRAPH)142,142,144
142 WRITE(6,146) OMEGMN,OMEGMX,XS
    GO TO 200
143 IF((IGRAPH.LT.0).OR.(IGRAPH.GT.1)) GO TO 142
144 WRITE(6,145)NUMDEC,YLOGMN
145 FORMAT(10X,'WITH A SPAN OF',I2,' DECADES,STARTING AT',F10.4,//,30X
   &,'*****NOTE*****',//,10X,'FOR THIS LOG PLOT, IT WILL BE NECESSARY
   &TO SHIFT THE ORIGIN TO THE LOWER RIGHT HAND CORNER,',//,10X,'WITH
   &THE ABCISSA RUNNING UP THE RIGHT HAND EDGE, AND THE ORDINATE INCRE
   &ASING TOWARDS THE LEFT ALONG THE BOTTOM'//)
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 146 FORMAT(10X,'FROM THE INITIAL VALUE OF',1PE10.3,'TO THE FINAL VALUE
    & OF',1PE10.3,' WITH A SCALE OF ',1PE10.3,' RADIANS PER INCH',/)
C*********************MAIN COMPUTATIONAL SECTION DO LOOPS****************
 200 IF((IWRITE.LT.1).OR.(IWRITE.GT.900)) GO TO 203
     WRITE(6, 201)IRUN,NPARAM,IWRITE
 201 FORMAT(1H1,10X,'*********THE OUTPUT DATA FOR RUN NUMBER',I2,', PAR
    &AMETER NUMBER',I2,' IS**********',////,10X,I2,' COMPUTATIONS WILL
    &BE PERFORMED BETWEEN EACH PRINTOUT',///)
     IF(NGAMMA.GT.0) WRITE(6,202) ZGAMMA(IGAMMA)
 202 FORMAT(10X,'THE VALUE OF GAMMA, THE THIRD VARIABLE IS:',1PE10.3,/)
 203 DO 520 LOM=1,NLIMIT
     JCURVE=0
     DO 519 ICOEF=1,3,2
     ZSIGN=ICOEF-2
     IF((IWRITE.LT.1).OR.(IWRITE.GT.900)) GO TO 208
     WRITE(6,205 )LOM
 205 FORMAT(/,10X,'COMPUTATIONAL RESULTS FOR CURVE NUMBER',I3)
     IF (NPARAM-7)206,208,208
 206 WRITE(6,207 )ZSIGN
 207 FORMAT(1H+,55X,' USING A QUADRATIC FORMULA COEFFICIENT OF',F4.1,/)
 208 IP=0
     YMAGPK=0.0
     OMEGPK=0.0
     DO 518 INCRMT=1,900
     GO TO (209,216,253,216,212,254,257,1000),NPARAM
 209 OMEGA=ZOMEGA(LOM)
 211 CONST=ALPHMN+(INCRMT-1)*(ALPHMX-ALPHMN)/899.0
 212 IF(MODORD-2)260,213,213
 213 DO 214  I=1,LOE
     A(I)=HA(I)+HAT(I)*CONST
 214 B(I)=HBT(I)+HCT(I)*CONST
     DO 215 I =1,MOE
     D(I)=HD(I)+HDT(I)*CONST
 215 E(I)=HET(I)+HFT(I)*CONST
     GO TO 263
 216 IF(IMAG-1)217,218,219
 217 DUMMY1=10.0**(ZMAGSQ(LOM)/10.0)
     GO TO 220
 218 DUMMY1=ZMAGSQ(LOM)*ZMAGSQ(LOM)
     GO TO 220
 219 DUMMY1=ZMAGSQ(LOM)
 220 IF(NPARAM-2)211,211,212
 253 OMEGA=CONOME
 254 IF(MODORD-2)255,256,1000
 255 CONST=ZBETA(LOM)
     GO TO 260
 256 CONST=ZALPHA(LOM)
     GO TO 213
 257 IF(MODORD-2)259,258,258
 258 BETA=ZBETA(LOM)
     CONST=CONALP
     GO TO 213
 259 BETA=ZALPHA(LOM)
     CONST=CONBET
 260 DO 261  I=1,LOE
     A(I)=HA(I)+HBT(I)*CONST
 261 B(I)=HAT(I)+HCT(I)*CONST
     DO 262  I=1,MOE
     D(I)=HD(I)+HET(I)*CONST
 262 E(I)=HDT(I)+HFT(I)*CONST
 263 GO TO (350,350,275, 290,264 ,291,294,1000),NPARAM
 264 OMEGA=ZOMEGA(LOM)
 275 ZMAG=XMAGMN+(INCRMT-1)*(XMAGMX-XMAGMN)/899.0
     IF(IMAG-1)276,277,278
 276 DUMMY1=10.0**(ZMAG/10.0)
     GO TO 350
 277 DUMMY1=ZMAG*ZMAG
     GO TO 350
 278 DUMMY1=ZMAG
     GO TO 350
 290 IF(IGRAPH)292,292,294
 291 IF((IGRAPH.GE.0).AND.(IGRAPH.LE.1)) GO TO 294
 292 OMEGA=OMEGMN+INCRMT*(OMEGMX-OMEGMN)/900.0
     GO TO 350
 294 WINC=1.0+(INCRMT-1)*NUMDEC/899.0
     OMEGA=(YLOGMN*10.0**WINC)/10.0
C**************NUMERATOR COEFFICIENT CALCULATIONS***********************
 350 SNB2=0.0
     SNB1=0.0
```

```
      SNBO=0.0
      SUMN=0.0
      NDOUN=2*NORN
      DO 373 ILOOP=1,4
      DO 373 NP=2,LOE
      DO 373 MQ=1,NORN
      MP=MQ-1
      NQ=NP-1
      IF((NP+MQ)-NDOUN)361,361,373
  361 IF((MP+NQ)-NORN)362,362,373
  362 IF(NP-MP)373,373,363
  363 IF(MP)364,364,366
  364 C=1.0
      GO TO 367
  366 C=2.0
  367 GO TO (368,369,370,371),ILOOP
  368 CSNB2 = (C*B(NP+MP)*B(NP-MP))*OMEGA**(2*NQ)*(-1.0)**(MQ+1)
      SNB2 = SNB2 + CSNB2
      GO TO 373
  369 CSNB1 = (C*A(NP+MP)*B(NP-MP)+C*A(NP-MP)*B(NP+MP))*OMEGA**(2*NQ)*(-
     11.0)**(MQ+1)
      SNB1 = SNB1 + CSNB1
      GO TO 373
  370 CSNBO = (C*A(NP+MP)*A(NP-MP))*OMEGA**(2*NQ)*(-1.0)**(MQ+1)
      SNBO = SNBO + CSNBO
      GO TO 373
  371 COEFN = (C*A(NP+MP)*A(NP-MP)+C*A(NP+MP)*B(NP-MP)*BETA+C*A(NP-MP)*B
     1(NP+MP)*BETA+C*B(NP+MP)*B(NP-MP)*BETA*BETA)*OMEGA**(2*NQ)*(-1.0)**
     2(MQ+1)
      SUMN = SUMN + COEFN
  373 CONTINUE
      SNB2 = SNB2 + B(1)*B(1)
      SNB1 = SNB1 + 2.0*A(1)*B(1)
      SNBO=SNBO+A(1)*A(1)
      SUMN = SUMN + A(1)*A(1)+2.0*A(1)*B(1)*BETA+B(1)*B(1)*BETA*BETA
C************DENOMINATOR COEFFICIENT CALCULATIONS************************
      SDB2=0.0
      SDB1=0.
      SDBO=0.
      SUMD=0.
      NDOUD=2*NORD
      DO 385 ILOOP=1,4
      DO 385 NP=2,MOE
      DO 385 MQ=1,NORD
      MP=MQ-1
      NQ=NP-1
      IF((NP+MQ)-NDOUD)375,375,385
  375 IF((MP+NQ)-NORD)376,376,385
  376 IF(NP-MP)385,385,377
  377 IF(MP)378,378,379
  378 C=1.0
      GO TO 380
  379 C=2.0
  380 GO TO (381,382,383,384),ILOOP
  381 CSDB2 =  C*E(NP+MP)*E(NP-MP)*OMEGA**(2*NQ)*(-1.0)**(MQ+1)
      SDB2 = SDB2 + CSDB2
      GO TO 385
  382 CSDB1 = (C*D(NP+MP)*E(NP-MP)+C*D(NP-MP)*E(NP+MP))*OMEGA**(2*NQ)*(-
     11.0)**(MQ+1)
      SDB1 = SDB1 + CSDB1
      GO TO 385
  383 CSDBO = (C*D(NP+MP)*D(NP-MP))*OMEGA**(2*NQ)*(-1.0)**(MQ+1)
      SDBO = SDBO + CSDBO
      GO TO 385
  384 COEFD = (C*D(NP+MP)*D(NP-MP)+C*D(NP+MP)*E(NP-MP)*BETA+C*D(NP-MP)*E
     1(NP+MP)*BETA+C*E(NP+MP)*E(NP-MP)*BETA*BETA)*OMEGA**(2*NQ)*(-1.0)**
     2(MQ+1)
      SUMD = SUMD + COEFD
  385 CONTINUE
      SUMD = SUMD + D(1)*D(1)+2.0*D(1)*E(1)*BETA+E(1)*E(1)*BETA*BETA
      SDB2 = SDB2 + E(1)*E(1)
      SDB1 = SDB1 + 2.0*D(1)*E(1)
      SDBO = SDBO + D(1)*D(1)
      IF(NPARAM-7)413, 386,1000
C************************PARAM 7 PHASE COMPUTATIONAL SECTION************
  386 TOE=LOE/2.0
      PNEVEN=0.
      PDEVEN=0.
      PNODD=0.
```

```
      PDODD=0.
      JOE=LOE/2
      IF(JOE-TOE)387,388,388
  387 JON=(LOE+1)/2
      MON=JON
      GO TO 389
  388 JON=(LOE+2)/2
      MON=JON-1
      IF(NORN-1)393,389,389
  389 DO 390 I=2,JON
  390 PNODD=PNODD+(A(2*I-2)+B(2*I-2)*BETA)*OMEGA**(2*I-3)*(-1.0)**(I)
      IF(NORN-1)393,393,391
  391 DO 392 I=2,MON
  392 PNEVEN=PNEVEN+(A(2*I-1)+B(2*I-1)*BETA)*OMEGA**(2*I-2)*(-1.)**(I-1)
  393 PNEVEN=PNEVEN+A(1)+B(1)*BETA
      TOE=MOE/2.0
      JOE=MOE/2
      IF(JOE-TOE)394,395,395
  394 JON=(MOE+1)/2
      MON=JON
      GO TO 396
  395 JON=(MOE+2)/2
      MON=JON-1
      IF(NORD-1)400,396,396
  396 DO 397 I=2,JON
  397 PDODD=PDODD+(D(2*I-2)+E(2*I-2)*BETA)*OMEGA**(2*I-3)*(-1.0)**(I)
      IF(NORD-1)400,400,398
  398 DO 399 I=2,MON
  399 PDEVEN=PDEVEN+(D(2*I-1)+E(2*I-1)*BETA)*OMEGA**(2*I-2)*(-1.)**(I-1)
  400 PDEVEN=PDEVEN+D(1)+E(1)*BETA
      TANG=(PDEVEN*PNODD-PNEVEN*PDODD)/(PNEVEN*PDEVEN+PNODD*PDODD)
      RAD=ATAN(TANG)
      SINE=SIN(RAD)
      IF(TANG)402,402,401
  401 SINE=-SINE
  402 YMAGSQ=ABS(SUMN/SUMD)
      YMAG=SQRT(YMAGSQ)
      YMAGDB=20.0*.4342945*ALOG(YMAG)
      YMAGAB=ABS(YMAGDB)
      IF(YMAGPK-YMAGDB)403,404,404
  403 YMAGPK=YMAGDB
      OMEGPK=OMEGA
  404 IF(DBMAX-YMAGDB)405,406,406
  405 YMAGDB=DBMAX
  406 IF(YMAGDB-DBMIN)407,408,408
  407 YMAGDB=DBMIN
  408 IF((XS*8.0)-YMAGAB)409,410,410
  409 YMAGAB=8.0*XS
  410 GO TO 426
C*****************QUADRATIC FORMULA SOLUTION OF DEPENDENT VARIABLE,PARAM
  413 ACO=DUMMY1*SDB2-SNB2
  416 BCO=DUMMY1*SDB1-SNB1
      CCO=DUMMY1*SDB0-SNB0
      IF(ABS(ACO).LT.1.0E-35) GO TO 425
      DPP=BCO*BCO-4.0*ACO*CCO
      IF(DPP.LT.0.0) GO TO 425
      DSQR=ZSIGN*SQRT(DPP)
      BETA=(DSQR-BCO)/(2.0*ACO)
      IF(BETA-BETAMN)425,426,424
  424 IF(BETA-BETAMX)426,426,425
  425 IF(IP-1)500,505,507
  426 IF((IWRITE.LT.1).OR.(IWRITE.GT.900)) GO TO 450
C********************COMPUTED DATA PRINTOUT SECTION**********************
      ISTEP=ISTEP+1
      IF(IP)428,428,427
  427 IF(ISTEP-IWRITE)450,430,430
  428 WRITE(6,429)LOM
  429 FORMAT(/,10X,'    ALPHA        BETA      MAGNITUDE         OMEGA
     & TANGENT  ;  CURVE NUMBER',I2,/)
  430 IF(MODORD.EQ.2) GO TO 431
      GO TO (433,435,436,437,439,441,443,1000),NPARAM
  431 GO TO (432,434,445,438,440,442,444,1000),NPARAM
  432 WRITE(6,6)    CONST,BETA,CONMAG,OMEGA
      GO TO 448
  433 WRITE(6,6)    BETA,CONST,CONMAG,OMEGA
      GO TO 448
  434 WRITE(6,6)    CONST,BETA,ZMAGSQ(LOM),OMEGA
      GO TO 448
  435 WRITE(6,6)    BETA,CONST,ZMAGSQ(LOM),OMEGA
```

```
      GO TO 448
  436 WRITE(6,6)    BETA,CONST,ZMAG,OMEGA
      GO TO 448
  437 WRITE(6,6)    BETA,CONBET,ZMAGSQ(LOM),OMEGA
      GO TO 448
  438 WRITE(6,6)    CONALP,BETA,ZMAGSQ(LOM),OMEGA
      GO TO 448
  439 WRITE(6,6)    BETA,CONBET,ZMAG,OMEGA
      GO TO 448
  440 WRITE(6,6)    CONALP,BETA,ZMAG,OMEGA
      GO TO 448
  441 WRITE(6,6)    BETA,CONST,CONMAG,OMEGA
      GO TO 448
  442 WRITE(6,6)    CONST,BETA,CONMAG,OMEGA
      GO TO 448
  443 WRITE(6,6)   ZALPHA(LOM),CONBET,YMAGDB,OMEGA,TANG
      GO TO 448
  444 WRITE(6,6)   CONALP,ZBETA(LOM),YMAGDB,OMEGA,TANG
      GO TO 448
  445 WRITE(6,6)    CONST,BETA,ZMAG,OMEGA
  448 ISTEP=0
C********************GRAPHICAL ARRAY STORAGE SECTION*********************
  450 IP=IP+1
      Y(IP)= BETA
  452 GO TO (453,453,460,470,460,470,475 ,1000),NPARAM
  453 X(IP)= CONST
      GO TO 500
  460 X(IP)=ZMAG
      GO TO 500
  470 X(IP)=OMEGA
      IF(NPARAM-6)471,472,475
  471 IF(IGRAPH)500,500,474
  472 IF((IGRAPH.LT.0).OR.(IGRAPH.GT.1)) GO TO 500
  474 X(IP)=LX-BETA
      GO TO 479
  475 X(IP)=-YMAGDB
      U(LOM,IP)=U(LOM,IP)+YMAG
      V(LOM,IP)=(ALOG10(OMEGA/YLOGMN)*LY/NUMDEC)*YS
      S(LOM,IP)=SINE*YMAGAB/(XS*TANG)
      T(LOM,IP)=SINE*YMAGAB/XS
      IC(LOM)=IP
      IF(IQUAD)476,478,478
  476 W(LOM,IP)=-4.0*RAD/1.5708
      IF(RAD)477,479,479
  477 W(LOM,IP)=8.0-W(LOM,IP)
      GO TO 479
  478 W(LOM,IP)=XS*(LX/2.-IY)-RAD*XS*LX/3.1414
      IF((IP).LE.(1))GO TO 479
      IF((YMAGDB).NE.(DBMIN)) GO TO 479
      IF((W(LOM,IP)).GE.(W(LOM,IP-1)))GO TO 479
      W(LOM,IP)=W(LOM,IP-1)
      V(LOM,IP)=V(LOM,IP-1)
  479 Y(IP)=(ALOG10(OMEGA/YLOGMN)*LY/NUMDEC)*YS
  500 IF(INCRMT.LT.900) GO TO 518
C***************************GRAPHICAL CURVE PLOTTING**********************
      IF(IP-1)501,505,507
  501 IF(JCURVE.GT.0) GO TO 503
      WRITE(6,502) LOM
  502 FORMAT(10X,'NO GRAPH POINTS WERE FOUND FOR CURVE NUMBER ',I2,/)
      GO TO 518
  503 WRITE(6, 504)LOM
  504 FORMAT(10X,'NO FURTHER POINTS WERE FOUND FOR CURVE NO.',I3,/)
      GO TO 518
  505 WRITE(6,506)
  506 FORMAT(10X,'ONLY ONE POINT WAS FOUND, AND WILL NOT BE PLOTTED',/)
      GO TO 516
  507 J=LOM+50
      JCURVE=1
      IF(IGRAPH.LE.-2) GO TO 516
      IF((NPARAM.EQ.7).AND.(IPAR7.LT.NPAR7))GO TO 518
      IF((ZSIGN.GT.0.0).AND.(JCURVE.GT.0))J=50
      WRITE(6,511)LOM
  511 FORMAT(/,10X,'THE PREVIOUS POINTS WILL BE PLOTTED AS CURVE ',I2,/)
      IF(NPARAM-7)514,512,514
  512 WRITE(6,513)YMAGPK,OMEGPK
  513 FORMAT(/,10X,'THE MAXIMUM VALUE OF MAGNITUDE IS',F10.4,' LOCATED A
     &T OMEGA EQUAL TO',F10.4,/)
      J=50
  514 CALL DRAW(IP,X,Y,MCD,0,LA(J),ITITLE,XS,YS,IX,IY,2,2,LX,LY,1,LAST)
```

```
      MOD=2
  516 IP=0
  518 CONTINUE
      IF(NPARAM.EQ.7) GO TO 520
  519 CONTINUE
  520 CONTINUE
      IF((IPAR7.GE.NPAR7).OR.(NPARAM.NE.7)) GO TO 529
C*******GAMMA & PARAM-7 SUPERPOSITION : SEE INPUT DATA SYMBOL NGAMMA****
  517 IF((NGAMMA.LE.0).OR.(IGAMMA.GT.NGAMMA)) GAMMA=0.0
      IF(IPAR7.GT.1)GO TO 523
      DO 521 I=1,LOE
      OLDHA(I)=HA(I)
      OLDHAT(I)=HAT(I)
      OLDHBT(I)=HBT(I)
  521 OLDHCT(I)=HCT(I)
      DO 522 I=1,MOE
      OLDHD (I)=HD (I)
      OLDHDT(I)=HDT(I)
      OLDHET(I)=HET(I)
  522 OLDHFT(I)=HFT(I)
  523 IPAR7=IPAR7+1
      IF(NGAMMA.GT.0) GO TO 526
      READ(5,5) (HA(I),I=1,LOE)
      READ(5,5) (HAT(I), I=1,LOE)
      READ(5,5) (HBT(I), I=1,LOE)
      READ(5,5) (HCT(I), I=1,LOE)
      READ(5,5) (HD (I), I=1,MOE)
      READ(5,5) (HDT(I), I=1,MOE)
      READ(5,5) (HET(I), I=1,MOE)
      READ(5,5) (HFT(I), I=1,MOE)
      IF(IPAR7.LT.NPAR7) GO TO 200
      DO 525 LOM=1,NLIMIT
      IP=IC(LOM)
      DO 524 I=1,IP
      X(I)=-U(LOM,I)
  524 Y(I)=V(LOM,I)
      J=LOM+1
  525 CALL DRAW(IP,X,Y,MOD,0,LA(J),ITITLE,XS,YS,IX,IY,2,2,LX,LY,1,LAST)
  526 DO 527 I=1,LOE
      HA (I)=OLDHA (I)+GAMMA*GAMHA (I)
      HAT(I)=OLDHAT(I)+GAMMA*GAMHAT(I)
      HBT(I)=OLDHBT(I)+GAMMA*GAMHBT(I)
  527 HCT(I)=OLDHCT(I)+GAMMA*GAMHCT(I)
      DO 528 I=1,MOE
      HD (I)=OLDHD (I)+GAMMA*GAMHD (I)
      HDT(I)=OLDHDT(I)+GAMMA*GAMHDT(I)
      HET(I)=OLDHET(I)+GAMMA*GAMHET(I)
  528 HFT(I)=OLDHFT(I)+GAMMA*GAMHFT(I)
      IF(IGAMMA.LE.NGAMMA) GO TO 26
  529 IF((NPARAM.NE.7).OR.(IGRAPH.LE.-2)) GO TO 999
C***************************PARAM 7 PHASE CURVES**************************
      IPAR7=2
      IF(IPHASE)537,532,999
  530 DO 531 I=1,12
  531 ITITLE(I)=JTITLE(I)
  532 DO 536 LOM=1,NLIMIT
      IP=IC(LOM)
      DO 533 I=1,IP
      X(I)=W(LOM,I)
  533 Y(I)=V(LOM,I)
  536 CALL DRAW(IP,X,Y,MOD,0,LA(50),JTITLE,XS,1.,0,IY,2,2,LX,LY,1,LAST)
      IPAR7=1
  537 IF((IPOLAR).LE.(0)) GO TO 999
C***************************PARAM 7 POLAR PLOT CURVES*********************
      MOD=1
      DO 542 I=1,8
      RADIUS=I/2.0
      DO 539 J=1,360
      RADIAN=6.28318/360*J
      X(J)=RADIUS*COS(RADIAN)
  539 Y(J)=RADIUS*SIN(RADIAN)
      CALL DRAW(J,X,Y,MOD,0,LA(1),KTITLE,1.,1.,4,4,2,2,8,8,0,LAST)
  542 MOD=2
      DO 543 J=10,180,10
      X(1)=X(J)
      X(2)=-X(1)
      Y(1)=Y(J)
      Y(2)=-Y(1)
  543 CALL DRAW(2,X,Y,2,0,LA(1),KTITLE,1.,1.,4,4,2,2,8,8,0,LAST)
```

```
      DO 545 LOM=1,NLIMIT
      IP=IC(LOM)
      DO 544 J=1,IP
      X(J)=S(LOM,J)
  544 Y(J)=T(LOM,J)
      L=LOM+1
  545 CALL DRAW(IP,X,Y,2,0,LA(L),KTITLE,1.,1.,4,4,2,2,8,8,0,LAST)
  999 IF((MOD).NE.(2)) GO TO 1000
      X(1) = 0.0
      Y(1) = 0.0
      X(2) = (LX-IY)*XS
      Y(2) = 0.0
      CALL DRAW (2,X,Y,3,0,LA(50),ITITLE,1.0,1.0,0,0,2,2,LX,LY,1,LAST)
      MOD=1
      IF((IPAR7).NE.(1)) GO TO 55
 1000 CONTINUE
      STOP
      END
C*********SAMPLE DATA DECK**************************************************
       1
       4          1          2          4          6
       0          3          1          8          8          8          14
       0         10         -1          1          1                      9   10
 0.00047    0.036      12.0       10.
  -0.001    0.007        0.0        0.05     -3.0       21.0       0.0        80.
  -0.001      0.0     0.00047     0.002      0.003      0.004      0.006
   0.0      0.01      0.02      0.03       0.036      0.04       0.045
  -3.0      0.0      3.0         6.0        9.0       12.0       15.0         18.
   1.0       4.0       10.0       12.0      15.0       20.0       30.0         40.
  45.       50.       55.        60.        70.        80.
100000.0
(BLANK CARD)
(BLANK CARD)
(BLANK CARD)
100000.      500.0      60.0       1.0
                     100000.0
          100000.0
 (BLANK CARD)
GLAVIS, PARAM 1, 3RD ORDER T/A F.B., OMEGA CURVS
ORDINATE=ALPHA, ABCISSA=BETA, CONMAGNITUDE=12DB
GLAVIS, PARAM 2, 3RD ORDER T/A F.B., MAG CURVES
ORDINATE=ALPHA, ABCISSA=BETA, CONOMEGA=10RAD/SEC
GLAVIS, PARAM 4, 3RD ORDER T/A F.B., MAG CURVES
ORDINATE=ALPHA, ABCISSA=OMEGA, CONBETA=0.036
GLAVIS, PARAM 6, 3RD ORDER T/A F.B., BETA CURVES
ORDINATE=ALPHA, ABCISSA=OMEGA, CONMAGNITUDE=12DB
```

LOGIC OUTLINE OF PROGRAM PARAMS

1. Read number of runs submitted.
2. Set run number equal to 1.
3. Read input data for appropriate run.
4. Print input data for appropriate run, if requested.
5. Calculate graph scales and location of origin.
6. Set NPARAM equal to the first PARAM value.
7. Select initial values for appropriate PARAM.
8. Print graphical information for appropriate PARAM, if requested.
9. Construct logarithmic grid if required.
10. Set curve number equal to 1.
11. Select curve value for appropriate curve number.
12. Set abcissa variable to minimum allowable value.
13. Set number of graph points equal to 0.
14. Compute numerator and denominator polynomial coefficients.
15. Check for possible division by 0.
16. Calculate ordinate value, using the quadratic formula for PARAM-1 through PARAM-6.
17. Check if calculated ordinate value is within graph limits. If yes, continue; if no, go to statement 21.
18. Increase number of points by 1.
19. Print computed values.
20. Store values necessary plotting.
21. Increase abcissa variable by pre-computed increment.
22. Check if abcissa variable exceeds maximum graph limit. If yes, continue; if no, go to statement 14.

23. Check if any points were found within graph limits. If none, so state and continue without plotting. If one, so state and continue without plotting. If more than one, continue and plot the graph points.

24. Check if both positive and negative coefficients were used in the quadratic formula solution where applicable. If yes, continue; if no, change sign and return to statement 12.

25. Increase curve number by 1.

26. Check if curve number exceeds the maximum number of curves stipulated in input data. If yes, continue; if no, return to statement 11.

27. Draw graph and plot computed curve data on same.

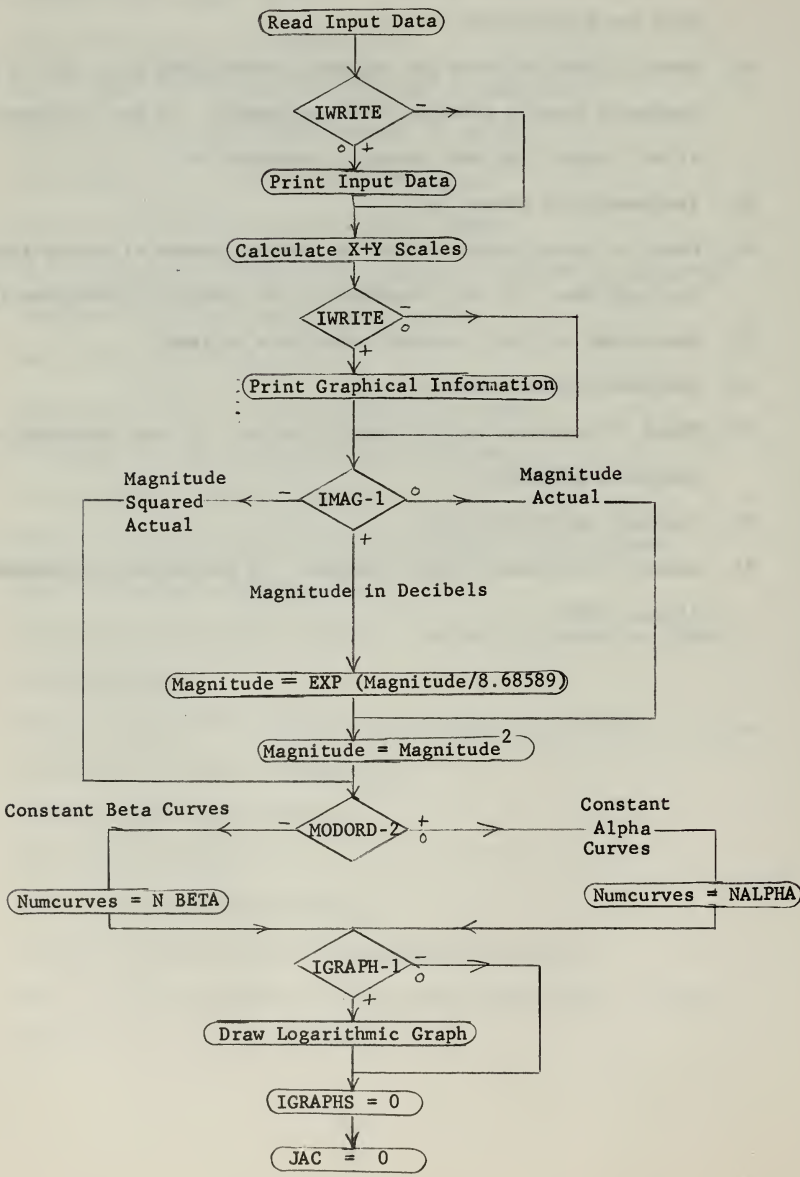
28. Increase parameter number by 1.

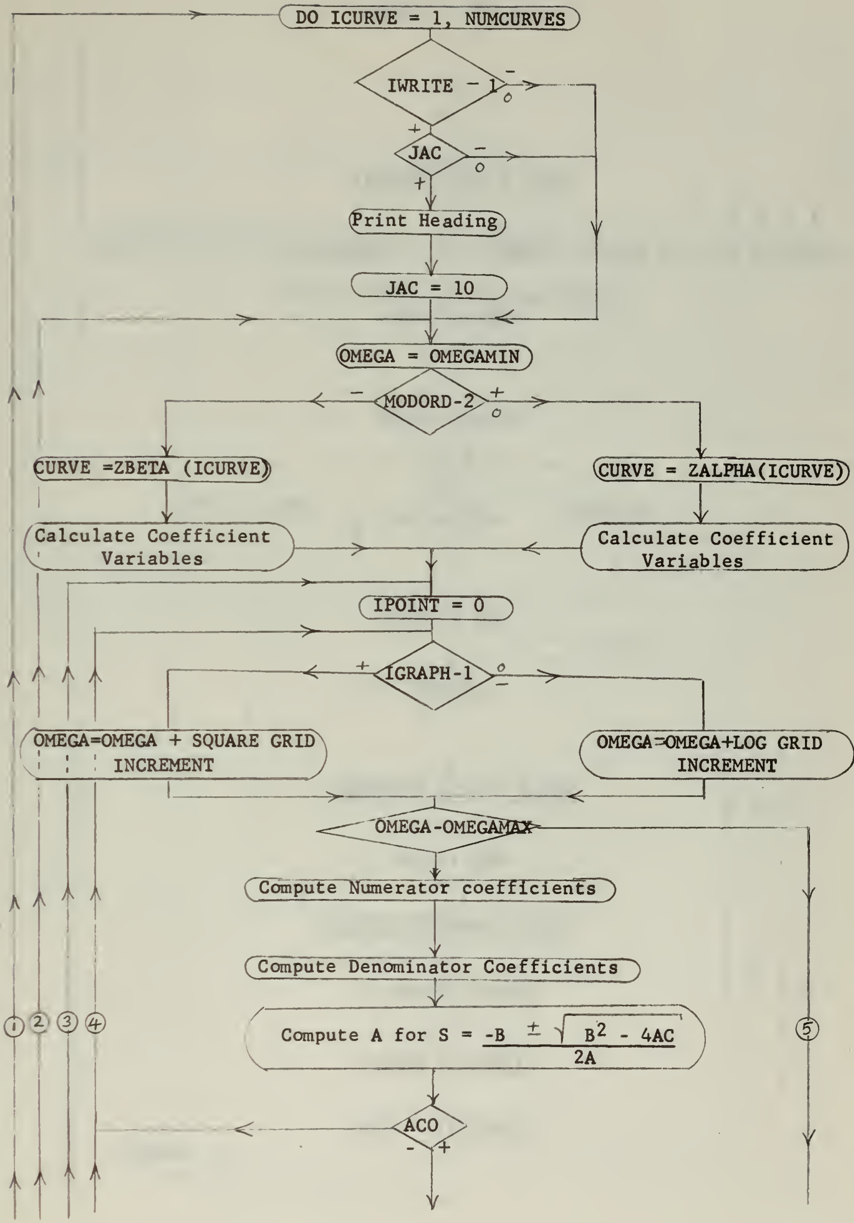
29. Check if parameter number exceeds maximum. If yes, continue; if no, return to statement 7.

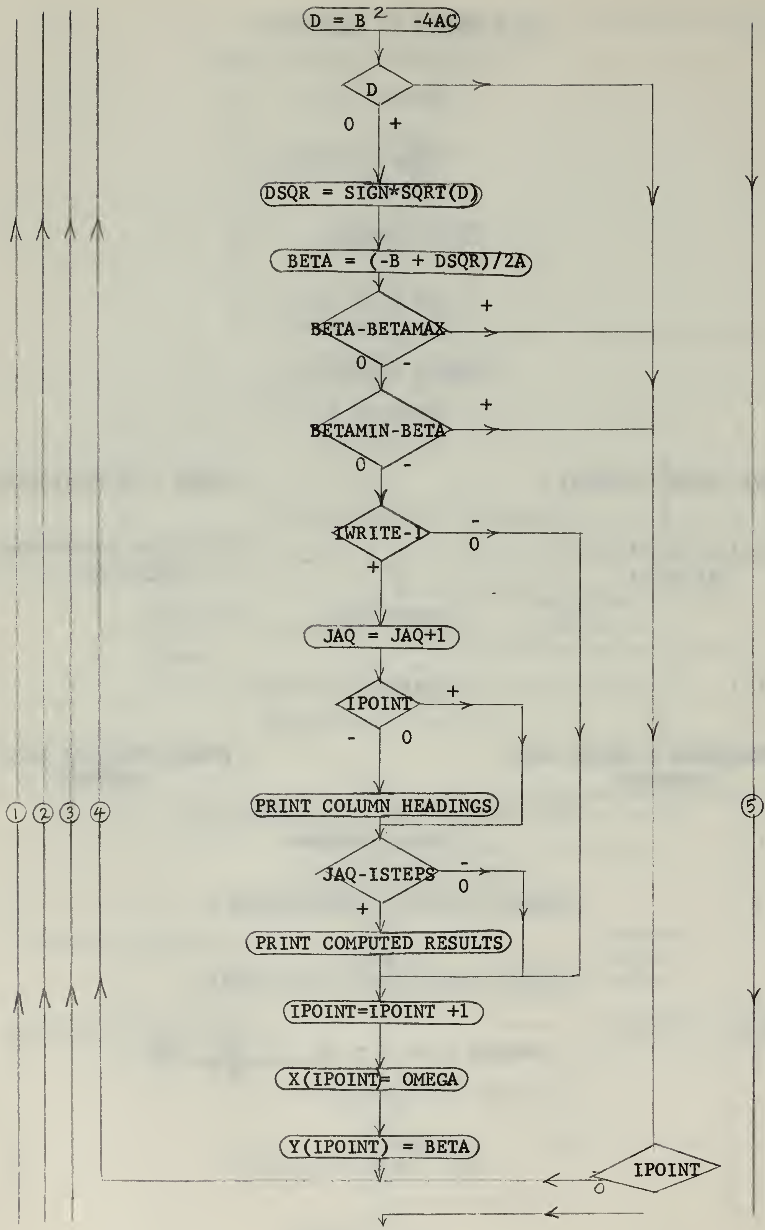
30. Increase run number by 1.

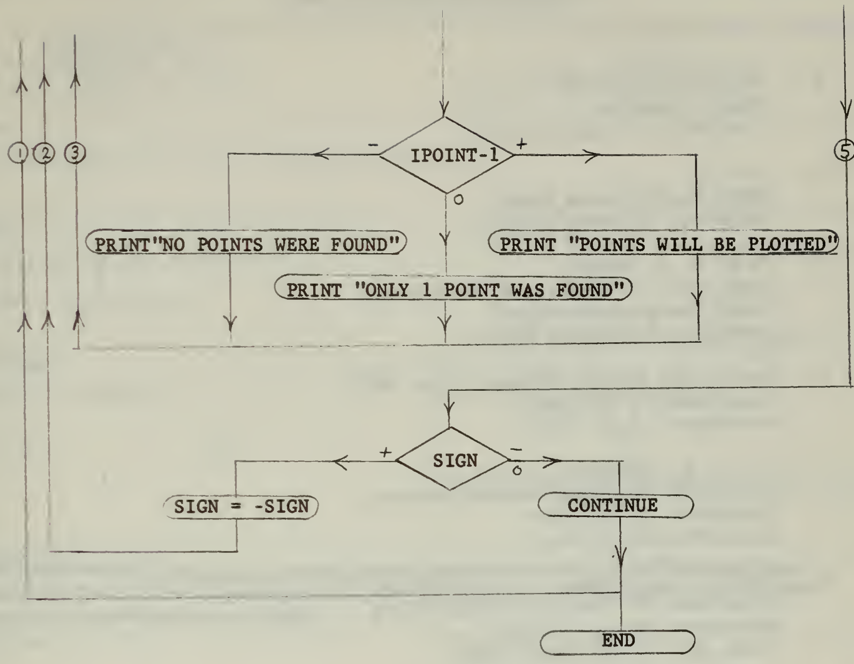
31. Check if run number exceeds maximum. If no, return to statement 3; if yes, stop.

FLOW CHART FOR PARAMETER SIX




DO ICURVE = 1, NUMCURVES
IWRITE - 1
JAC
Print Heading
JAC = 10
OMEGA = OMEGAMIN
MODORD-2
CURVE =ZBETA (ICURVE)
CURVE = ZALPHA(ICURVE)
Calculate Coefficient Variables
Calculate Coefficient Variables
IPOINT = 0
IGRAPH-1
OMEGA=OMEGA + SQUARE GRID INCREMENT
OMEGA=OMEGA+LOG GRID INCREMENT
OMEGA-OMEGAMAX
Compute Numerator coefficients
Compute Denominator Coefficients
Compute A for S = -B ± √(B² - 4AC) / 2A
ACO
1 2 3 4
5


D = B 2 -4AC
D
0
+
DSQR = SIGN*SQRT(D)
BETA = (-B + DSQR)/2A
BETA-BETAMAX
+
0
-
BETAMIN-BETA
+
0
-
IWRITE-1
-
0
+
JAQ = JAQ+1
IPOINT
+
-
0
PRINT COLUMN HEADINGS
JAQ-ISTEPS
-
0
+
PRINT COMPUTED RESULTS
IPOINT=IPOINT +1
X(IPOINT)= OMEGA
Y(IPOINT) = BETA
IPOINT
0
1
2
3
4
5



IPOINT-1
PRINT "NO POINTS WERE FOUND"
PRINT "ONLY 1 POINT WAS FOUND"
PRINT "POINTS WILL BE PLOTTED"
SIGN
SIGN = -SIGN
CONTINUE
END

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13. ABSTRACT

Algebraic Methods are employed for frequency response studies of physical systems. The transfer function of each system's mathematical model is composed of at least two independent variables embedded in the coefficients. These, together with magnitude, angular frequency, and phase angle are the variable parameters to be studied.

Computer programs are developed for graphically representing these variable parameters. Interpretation of these results is made to evaluate the utility of such methods in design and analysis studies.

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